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MARCH, 1860.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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CONTENTS.

MARCH, 1860.

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THE

MATHEMATICAL MONTHLY.

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PRIZE PROBLEMS FOR STUDENTS.

I. Find x from the equation,

$$\sqrt[5]{a+x} + \sqrt[5]{a-x} = b,$$

by quadratics.

II. If a circle be described touching the base of a triangle and the sides produced, and a second circle be inscribed in the triangle; prove that the points where the circles touch the base are equidistant from its extremities, and that the distance between the points where they touch either of the sides is equal to the base.

III. Inscribe the maximum rectangle between the conchoid and its directrix. — Communicated by Prof. Daniel Kirkwood.

IV. Given a cask containing a gallons of wine. Through a cock at the bottom of the cask wine flows out at the rate of b gallons per minute, and through a hole at the top water flows in at the same rate. Supposing the water, as fast as it flows in, to mingle perfectly with the wine, how long before the quantities of wine and water in the cask will be equal? and how much wine will be left in the cask at the end of t minutes? — Communicated by Prof. C. A. Young.

V. Two circles being given in a plane, find geometrically the locus of the points from which chords of similar arcs in the two circles will be seen under the same angle, the chords being perpendicular

to the lines of vision drawn through the centres of the given circles. — Communicated by Prof. Wm. Chauvenet.

The solutions of these Problems must be received by the 1st of May, 1860.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. III., Vol. II.

The first Prize is awarded to John Q. Holliston, Sophomore Class, Hamilton College, Clinton, N. Y.

The second Prize is awarded to F. E. Tower, Senior Class, Amherst College, Amherst, Mass.

The third Prize is awarded to Frank N. Devereux, Boston, Mass.

Prize Solution of Problem I.

By Frank N. Devereux, Boston, Mass.

If two circles touch each other, any straight line passing through the point of contact cuts off similar parts of their circumferences.

The line joining the centres C and C' will pass through the point of contact B. Let A and A' be the points in which the line passing through the point of contact meets the circumferences. Join A and C, A' and C'. The triangles $A\,B\,C$ and $A'\,B\,C'$ thus formed are isosceles and similar, as is easily seen. The angles at the centres, $A\,C\,B$ and $A'\,C'\,B$, are therefore equal, and are measured by similar parts of the circumferences. Hence the proposition is true. This proposition applies whether the circles are tangent externally or internally, a fact not noticed by any of the competitors.

Prize Solution of Problem II.

Find the four roots of the recurring equation

$$x^4 - \tfrac{5}{2}\,x^3 + 2\,x^2 - \tfrac{5}{2}\,x + 1 = 0.$$

Dividing the given equation by x^2 it becomes

(1) $$(x^2 + \frac{1}{x^2}) - \tfrac{5}{2}(x + \frac{1}{x}) + 2 = 0.$$

Put $x + \frac{1}{x} = y$; then squaring, $x^2 + \frac{1}{x^2} = y^2 - 2$.

By substitution we get from (1)

$$y^2 - \tfrac{5}{2}y = 0. \qquad \therefore y = 2\tfrac{1}{2} \text{ or } 0.$$

$$\therefore x + \frac{1}{x} = 2\tfrac{1}{2}, \text{ from which } x = 2 \text{ or } \tfrac{1}{2}.$$

$$\therefore x + \frac{1}{x} = 0, \text{ from which } x = \pm\sqrt{-1}.$$

Or thus, the given equation may be written

$$x^2(x^2 - \tfrac{5}{2}x + 1) + x^2 - \tfrac{5}{2}x + 1 = 0;$$

or, $$(x^2 + 1)(x^2 - \tfrac{5}{2}x + 1) = 0.$$

$\therefore x^2 + 1 = 0$; or $x = \pm\sqrt{-1}$. $\therefore x^2 - \frac{5}{2}x + 1 = 0$; or $x = 2$ or $\frac{1}{2}$.

All the competitors gave one or the other of the above solutions.

Prize Solution of Problem III.

By John Q. Holliston, Hamilton College, Clinton, N. Y.

If $2\cos\theta = u + \frac{1}{u}$, prove that $2\cos 2\theta = u^2 + \frac{1}{u^2}$, $2\cos 3\theta = u^3 + \frac{1}{u^3}$ ..... $2\cos n\theta = u^n + \frac{1}{u^n}$; and then find the sum of the series, $\cos\theta + \cos 2\theta + \cos 3\theta$ ..... $+ \cos n\theta$.

By Euler's formula, and the problem,

$$2\cos\theta = e^{\theta\sqrt{-1}} + \frac{1}{e^{\theta\sqrt{-1}}} = u + \frac{1}{u}.$$

$$\therefore u = e^{\theta\sqrt{-1}}, \ \frac{1}{u} = \frac{1}{e^{\theta\sqrt{-1}}}. \qquad \therefore u^n = e^{n\theta\sqrt{-1}}, \ \frac{1}{u^n} = \frac{1}{e^{n\theta\sqrt{-1}}}.$$

Hence, and by Euler's formula,

$$e^{n\theta\sqrt{-1}} + \frac{1}{e^{n\theta\sqrt{-1}}} = u^n + \frac{1}{u^n} = 2\cos n\theta.$$

Denoting by Σ_n the sums relatively to n, we have

$$2\,\Sigma_n \cos n\theta = \Sigma_n u^n + \Sigma_n \frac{1}{u^n} = \frac{u^{n+1}-u}{u-1} + \frac{\frac{1}{u^{n+1}} - \frac{1}{u}}{\frac{1}{u} - 1};$$

since $u + u^2 + \&c.$, and $\frac{1}{u} + \frac{1}{u^2} + \&c.$, are geometrical series with ratios u and $\frac{1}{u}$.

Remark. — By dividing the terms of the first fraction, and multiplying those of the second, by $u^{\frac{1}{2}}$, we get

$$\Sigma_n \cos n\theta = \frac{u^{n+\frac{1}{2}} - u^{\frac{1}{2}} - \frac{1}{u^{n+\frac{1}{2}}} + \frac{1}{u^{\frac{1}{2}}}}{2\left(u^{\frac{1}{2}} - \frac{1}{u^{\frac{1}{2}}}\right)},$$

$$= \frac{u^{n+\frac{1}{2}} - \frac{1}{u^{n+\frac{1}{2}}} - \left(u^{\frac{1}{2}} - \frac{1}{u^{\frac{1}{2}}}\right)}{2\left(u^{\frac{1}{2}} - \frac{1}{u^{\frac{1}{2}}}\right)} = \frac{\sin\left(n+\frac{1}{2}\right)\theta - \sin\frac{1}{2}\theta}{2\sin\frac{1}{2}\theta}.$$

This form of the result was not given by any of the competitors.

Prize Solution of Problem IV.

John Q. Holliston, Hamilton College, Clinton, N. Y.

Having given the Right Ascensions and Declinations of two stars, to find the formula for the distance between them. Also, find what the distance becomes, when for one star A. R. is $8^h\ 12^m\ 38^s.17$, and Dec. $17°\ 23'\ 49''.8$ north, and for the other A. R. is $13^h\ 28^m\ 19^s.92$, and Dec. $21°\ 12'\ 37''.2$ south.

Let δ and δ' be the declinations of the two stars; then $90° - \delta$ and $90° - \delta'$ will be their co-declinations; and since the right ascensions are measured on the equator, their difference will measure the angle at the pole made by the meridians passing through the two stars. Let H denote this angle, and Δ the distance sought. We have then a spherical triangle in which the sides $90° - \delta$ and $90° - \delta'$ include the angle H, and Δ, the third side, may be found from the formula

$\cos \Delta = \cos(90^\circ - \delta)\cos(90^\circ - \delta') + \sin(90^\circ - \delta)\sin(90^\circ - \delta')\cos H,$

(1) $\quad = \sin\delta \sin\delta' + \cos\delta \cos\delta' \cos H,$

(2) $\quad = \sin\delta \cos\delta' \left(\frac{\sin\delta'}{\cos\delta'} + \cot\delta \cos H\right).$

Assume

(3) $$\cot\delta \cos H = \tan\omega = \frac{\sin\omega}{\cos\omega};$$

then, substituting, (2) becomes

(4) $$\cos\Delta = \frac{\sin\delta \sin(\delta' + \omega)}{\cos\omega}.$$

Formulas (3) and (4) need only tables of logarithmic sines, cosines, &c.; and we find $\Delta = 86^\circ\ 24'\ 12''.2$.

Prize Solution of Problem V.

By Asher B. Evans, Madison University, Hamilton, N. Y.

In a frustum of any pyramid or cone, the area of a section, parallel to the two bases and equidistant from them, is the arithmetical mean of the arithmetical and geometrical means of the areas of the two bases.

The three sections are evidently similar figures; hence their areas will be as the squares of any homologous lines. Let x, $x - y$, and $x + y$, represent any homologous lines in the upper base, middle section, and lower base, respectively. Then their areas may be represented by $m(x-y)^2$, mx^2, $m(x+y)^2$, respectively. The arithmetical mean of the two bases is

$$\frac{m(x-y)^2 + m(x+y)^2}{2} = m(x^2 + y^2);$$

their geometrical mean is

$$\sqrt{m^2(x-y)^2(x+y)^2} = m(x^2 - y^2);$$

and the arithmetical mean of $m(x^2 + y^2)$ and $m(x^2 - y^2)$ is mx^2, as was to be shown.

Simon Newcomb.
W. P. G. Bartlett.
Truman Henry Safford.

NOTES AND QUERIES.

1. What fraction is that, to the numerator of which if 1 be added, its value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

The successive multiples of $\frac{1}{3}$ are $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$, $\frac{6}{18}$, &c.; hence, taking unity from the numerator of each of these, we have the series

$$\frac{1}{6},\ \frac{2}{9},\ \frac{3}{12},\ \frac{4}{15},\ \frac{5}{18},\ \frac{6}{21},\ \&c.,$$

and the required fraction must necessarily be one of this series.

Again, the successive multiples of $\frac{1}{4}$ are $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$, $\frac{6}{24}$, &c.; whence taking unity from the denominator of each of these, we get

$$\frac{2}{7},\ \frac{3}{11},\ \frac{4}{15},\ \frac{5}{19},\ \frac{6}{23},\ \frac{7}{27},\ \&c.,$$

a series of fractions in which the required fraction must necessarily be found.

Examining these two series, we find that the third fraction in the second series is the same as the fourth fraction in the first series, and therefore, as the sought fraction must be found in both series, we conclude with the utmost certainty that $\frac{4}{15}$ is the fraction fulfilling the conditions of the question.

2. What fraction is that, whose numerator being doubled, and denominator increased by 7, the value becomes $\frac{2}{3}$; but the denominator being doubled, and the numerator increased by 2, the value becomes $\frac{3}{5}$?

Reasoning as in the preceding example, we must first take the successive multiples of $\frac{2}{3}$; divide the numerator of each multiple by 2, and decrease each denominator by 7; then we have the following series, viz.:—

$$\frac{3}{2},\ \frac{4}{5},\ \frac{5}{8},\ \frac{6}{11},\ \frac{7}{14},\ \frac{8}{17},\ \&c.$$

Again, take the consecutive multiples of $\frac{3}{5}$, decrease the numera-

tor of each by 2, and divide the denominator by 2; then we have the series

$$\tfrac{2}{5},\ \tfrac{4}{5},\ \tfrac{15}{16},\ \tfrac{10}{10},\ \tfrac{26}{25},\ \tfrac{16}{15},\ \&c.$$

The required fraction must exist in both series, in order that the two conditions in the enunciation may be complied with, and as $\frac{4}{5}$ is the only fraction common to both, we conclude that $\frac{4}{5}$ will satisfy the conditions stated in the question. — Prof. W. Rutherford, Woolwich. — *The Northumbrian Mirror,* 1838.

1. *Isochronous Motions.* — The principle of isochronous motions, as exhibited in the small oscillations of the pendulum, in the oscillations of the hair-spring balance, and in the descent of weights on the cycloid, is not usually set forth in the analytical treatment of these examples with sufficient prominence to be distinctly generalized and abstractly comprehended, though the analysis necessarily involves it.

In the following theorem and demonstration this principle is distinctly presented, and from it may be deduced all the cases that have been separately investigated.

Theorem. — *If a material point, constrained to move in any given path, tend to approach a given fixed point in the path, urged along by a force proportional to the length of the path between it and the given point; it will pass from a state of rest at any point whatever to the given point always in the same time; or it will pass from the given point with any velocity whatever to a state of rest in the same time.*

Demonstration. — If two material points start at rest in two positions on the path, and if we suppose that the two lengths of the path included between the given fixed point and these positions be divided into the same number of very small parts, the ratio of the corresponding parts, and the ratio of their distances from the given fixed point along the path, will be equal to the ratio of the lengths themselves; hence, the forces acting in corresponding parts will be in the ratio of the parts themselves; hence, the sum of all the

actions of the forces through any number of parts in one length will be to the sum of the actions of the forces in the same number of corresponding parts in the other length in the same ratio; hence, the two velocities which the two material points will have in passing through corresponding parts will be in the ratio of the parts themselves, and these corresponding parts will therefore be passed over in the same time, and any number of them in one length will be passed over in the same time as the same number of the corresponding ones in the other length; hence, the whole lengths will be described in the same time; that is, the two material points will arrive at the given fixed point at the same instant, and with velocities proportional to the lengths of the path they have described. If a material point set out from the given fixed point with any velocity, it is obvious that it will come to rest at that point from which it would attain this velocity in moving back to the given fixed point; and that it will have the same velocities in every part as in moving from rest to the given fixed point, though in a contrary direction; hence the time will be the same.

Since a body, descending upon any path under the action of gravity, is impelled along the path by the force $g \sin \tau$, in which g is the direct force of gravity, and τ the angle made by the direction of the curve at any point with the horizon, it follows, that the times of descent will be the same from all parts of the curve to the point for which $\tau = 0$, or the lowest point of the curve, if the arc s, reckoned from this lowest point, be proportional to the force $g \sin \tau$. The equation $s = 4 R \sin \tau$ is an equation of the cycloid, a curve which is therefore called the *Tautochrone.* In the common pendulum, the force $g \sin \tau$, which impels the ball along its circular path, is nearly proportional to the path itself when the amplitudes of the vibrations are small. The small motions of the pendulum are therefore nearly isochronous. The small motions of

a body vibrating upon the lowest portion of any curve are, for the same reason, nearly isochronous. When the force of the hair-spring of the watch-balance is proportional to the angle by which it is drawn from its position of equilibrium, the motions of the balance are, by the reasoning above, also isochronous. — W.

2. *Problem.** Given $A = \dfrac{15\,(E - \sin E)}{9\,E + \sin E}$ (1), and $T = \tan^2 \frac{1}{2} E$, to find

$$(2) \qquad A = \frac{T - \frac{6}{5}\,T^2 + \frac{9}{7}\,T^3 - \frac{12}{9}\,T^4 + \frac{15}{11}\,T^5 - \text{\&c.}}{1 - \frac{6}{15}\,T + \frac{7}{25}\,T^2 - \frac{8}{35}\,T^3 + \frac{9}{45}\,T^4 - \text{\&c.}}.$$

Dividing both terms of the value of A in (1) by $\cos E$ it becomes

$$(3) \qquad A = \frac{15\,(E \sec E - \tan E)}{9\,E \sec E + \tan E}.$$

Now, $\qquad \sec E = \dfrac{1 + \tan^2 \frac{1}{2} E}{1 - \tan^2 \frac{1}{2} E}$, and $\qquad \tan E = \dfrac{2 \tan \frac{1}{2} E}{1 - \tan^2 \frac{1}{2} E}$.

Substituting these values in (3) we get

$$(4) \quad A = \frac{30\,\{\frac{1}{2} E\,(1 + \tan^2 \frac{1}{2} E) - \tan E\}}{18\,\frac{1}{2} E\,(1 + \tan^2 \frac{1}{2} E) + 2 \tan \frac{1}{2} E} = \frac{30\,\{\frac{1}{2} E\,(1 + T) - T^{\frac{1}{2}}\}}{18\,\frac{1}{2} E\,(1 + T) + 2\,T^{\frac{1}{2}}}.$$

But, $\frac{1}{2} E = \tan^{-1} T^{\frac{1}{2}} = T^{\frac{1}{2}} - \frac{1}{3}\,T^{\frac{3}{2}} + \frac{1}{5}\,T^{\frac{5}{2}} - \frac{1}{7}\,T^{\frac{7}{2}} + \frac{1}{9}\,T^{\frac{9}{2}} - \text{\&c.}$; multiplying this value of $\frac{1}{2} E$ by $1 + T$, and subtracting $T^{\frac{1}{2}}$ from the product, the numerator in (4) becomes

$$30\,(\tfrac{2}{3}\,T^{\frac{3}{2}} - \tfrac{2}{15}\,T^{\frac{5}{2}} + \tfrac{2}{35}\,T^{\frac{7}{2}} - \tfrac{2}{63}\,T^{\frac{9}{2}} + \text{\&c.});$$

and by a similar process the denominator becomes

$$20\,(T^{\frac{1}{2}} + \tfrac{3}{5}\,T^{\frac{3}{2}} - \tfrac{3}{25}\,T^{\frac{5}{2}} + \tfrac{9}{175}\,T^{\frac{7}{2}} - \text{\&c.})$$

$$\therefore\; A = \frac{T^{\frac{3}{2}} - \frac{1}{5}\,T^{\frac{5}{2}} + \frac{3}{35}\,T^{\frac{7}{2}} - \frac{1}{21}\,T^{\frac{9}{2}} + \text{\&c.}}{T^{\frac{1}{2}} + \frac{3}{5}\,T^{\frac{3}{2}} - \frac{3}{25}\,T^{\frac{5}{2}} + \frac{9}{175}\,T^{\frac{7}{2}} - \text{\&c.}}.$$

If now we divide both terms of this value of A by $T^{\frac{1}{2}} + T^{\frac{3}{2}}$, we shall obtain (2) as was to be done. — Richard Cotter, Professor of Mathematics in Baltimore College.

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* From Gauss's *Theoria Motus.* See Davis's translation, page 46.

3. The series (10) page 87, Vol. I., taken *without limitation*, is always equal to unity. For, putting $\frac{y}{l} = \frac{x-1}{x}$, it becomes

$$n\left(\frac{x-1}{x}\right)^{n-1} - \frac{n(n-1)}{2!}\left(\frac{x-2}{x}\right)^{n-1} + \&c. = S;$$

but in the Calculus of Finite Differences we have

$$\Delta^n u^m = (u+n)^m - n(u+n-1)^m + \frac{n(n-1)}{2!}(u+n-2)^m - \&c.;$$

hence,

$$x^{n-1}(1-S) = \Delta^n (x-n)^{n-1} = 0. \qquad \therefore S = 1.$$

The special case of S, on page 87, was put equal to unity only because it expressed the *probability of a certainty*. We now deduce the same result from its algebraic form alone. — B.

4. Solve the differential equation

$$n^2 \varphi(r) + \frac{\varphi'(r)}{r} + \varphi''(r) = 0.$$

— Airy's *Tides and Waves*. Ency. Metrop.

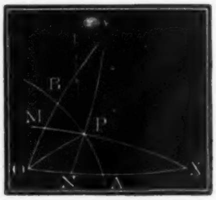
ON SPHERICAL ANALYSIS.

By George Eastwood, Saxonville, Mass.

[Continued from Page 166.]

Proposition II.

Knowing the co-ordinates, ON, OM, *of a point* P, *referred to oblique axes, to find its distance,* OP, *from the origin.*



We have already found, (1) and (4),

$$\tan OP = \tan r = \frac{\tan x}{\sin \varphi} = \frac{\sin \omega \tan y}{\sin \varphi};$$

but equation (3), viz.

$$\frac{\tan y}{\tan x} = \frac{\sin \varphi}{\sin(\omega - \varphi)},$$

gives

$$\tan\varphi = \frac{\sin\omega \tan y}{\tan x + \cos\omega \tan y},$$

$$\sin\varphi = \frac{\sin\omega \tan y}{(\tan^2 x + \tan^2 y + 2\tan x \tan y \cos\varphi)^{\frac{1}{2}}}.$$

The substitution of this value of $\sin\varphi$ in $\tan r$ gives

(10) $$\tan^2 r = \tan^2 x + \tan^2 y + 2\tan x \tan y \cos\omega.$$

If the axes be rectangular, then $\omega = \frac{1}{2}\pi$, and (10) reduces to

(11) $$\tan^2 r = \tan^2 x + \tan^2 y.$$

Cor. 1. In a plane we have

(12) $$r^2 = x^2 + y^2 + 2xy\cos\omega,$$

when the axes are oblique; and

(13) $$r^2 = x^2 + y^2,$$

when the axes are rectangular. Hence, if we agree, in the present and subsequent investigations, to designate the tangent-function of all great circle arcs emanating from the origin, by the symbols of those arcs, we have a perfect type of (12) and (13), in (10) and (11).

Cor. 2. If, with centre O and radius $OP =$ constant, we describe a less circle of the sphere, then

(14) $$r^2 = x^2 + y^2 + 2xy\cos\omega,$$

will be its equation for oblique, and

(15) $$r^2 = x^2 + y^2,$$

for rectangular axes, the origin being at the centre.

Proposition III.

To find the equation of a great circle which intersects the axes at given distances from the origin.

Let A and B be the points in which the great circle cuts the axes of reference. Through any point P in this circle, draw the project-

ing arcs XPM, YPN, and designate OA by α, OB by β, and the other arcs and angles as in Prop. I. Then the triangle, AOB, cut by the spherical transversal MPX, gives the relations

$$\sin BM : \sin BP = \sin \angle P : \sin \angle M,$$
$$\sin OX : \sin OM = \sin \angle M : \sin \angle X,$$
$$\sin AP : \sin AX = \sin \angle X : \sin \angle P.$$

In like manner, the triangle AOB, cut by the spherical transversal NPY, gives

$$\sin AN : \sin AP = \sin \angle P : \sin \angle N,$$
$$\sin OY : \sin ON = \sin \angle N : \sin \angle Y,$$
$$\sin BP : \sin BY = \sin \angle Y : \sin \angle P.$$

Compounding these respective groups of ratios, effacing common factors, and remembering that OX and OY are quadrantal arcs of great circles, we have the further relations

$$\sin BM \,.\, \sin AP = \sin BP \,.\, \sin OM \,.\, \sin AX,$$
$$\sin AN \,.\, \sin BP = \sin AP \,.\, \sin ON \,.\, \sin BY,$$

which, when expressed in symbols, give

$$\frac{\sin(\beta - y) \,.\, \sin AP}{\sin y \,.\, \cos \alpha \,.\, \sin BP} = 1,$$

$$\frac{\sin(\alpha - x) \,.\, \sin BP}{\sin x \,.\, \cos \beta \,.\, \sin AP} = 1.$$

Multiplying these equations together, member by member, and reducing, we obtain

$$\frac{\frac{\sin y}{\cos y}}{\frac{\sin \beta}{\cos \beta}} + \frac{\frac{\sin x}{\cos x}}{\frac{\sin \alpha}{\cos \alpha}} = 1,$$

or,

(16) $$\frac{y}{\beta} + \frac{x}{\alpha} = 1.$$

(17) $$\therefore y = -\frac{\beta}{\alpha} x + \beta.$$

As (1) and (2) are independent of ω, either of them may be regarded as the general equation of a great circle of the sphere referred to any axes.

When the axes are rectangular, and when OP is perpendicular to AB, then the right triangles BPO, APO give

$$\tan OP = \tan OB \,.\, \cos BOP = \beta \sin \varphi,$$
$$\tan OP = \tan OA \,.\, \cos AOP = \alpha \cos \varphi.$$
$$\therefore\ \tan \varphi = \frac{\alpha}{\beta} = \frac{1}{\frac{\beta}{\alpha}}.$$

Hence, by (7), the equation of OP is

(18) $$y = \frac{1}{\frac{\beta}{\alpha}}\, x.$$

Cor. 1. In a plane, the equation of a straight line, referred to rectangular axes, is

$$y = t\,x + b,$$

and the equation of a line perpendicular to this, from the origin, is

$$y = -\frac{1}{t}\, x.$$

If, therefore, we put $\tau = -\frac{\beta}{\alpha}$, (17) and (18) become

(19) $$y = \tau\, x + \beta,$$

(20) $$y = -\frac{1}{\tau}\, x.$$

Cor. 2. We may give to (19) this form,

(21) $$y = \tau\, x \pm \varrho\, (1 + \tau^2)^{\frac{1}{2}}.$$

For, if we assume $\varrho = OP$, then

$$\varrho = \beta \cos BOP,$$
$$= \pm \frac{\beta}{(1 + \tau^2)^{\frac{1}{2}}},$$

and

$$\beta = \pm\, \varrho\, (1 + \tau^2)^{\frac{1}{2}}.$$

Remark.—In the diagram, the great circle is represented as cutting the axis of x *east* of the origin, and hence α is counted positive. If it cut the axis of x *west* of the origin, then α must be counted negative, just as in a plane. Hence the sign of τ, in (19), (20), and (21), must be always counted the reverse of that of α.

NOTES ON ANALYTIC TRIGONOMETRY.

By O. Root, Professor of Mathematics in Hamilton College, Clinton, N. Y.

Let n be any number, and φ any arc whatever with a radius of unity; from the differential calculus we shall have

(1) $$d \cos \varphi = -\sin \varphi \, d\varphi,$$

(2) $$d \cos n\varphi = -n \sin n\varphi \, d\varphi.$$

From these equations we readily get

(3) $$n \frac{d \cos \varphi}{\sin \varphi} = \frac{d \cos n\varphi}{\sin n\varphi}.$$

And since

(4) $\sin \varphi = (1 - \cos^2 \varphi)^{\frac{1}{2}}$, and $\sqrt{(-1)} \sin \varphi = (\cos^2 \varphi - 1)^{\frac{1}{2}}$,

we may write (3) in the following form,

(5) $$n \frac{d \cos \varphi}{(\cos^2 \varphi - 1)^{\frac{1}{2}}} = \frac{d \cos n\varphi}{(\cos^2 n\varphi - 1)^{\frac{1}{2}}}.$$

Integrating (5) we shall get

$$n \log (\cos \varphi + \sqrt{(-1)} \sin \varphi) = \log (\cos n\varphi + \sqrt{(-1)} \sin n\varphi),$$

and this may be written

(6) $(\cos \varphi + \sqrt{(-1)} \sin \varphi)^n = (\cos n\varphi + \sqrt{(-1)} \sin n\varphi)$,

which is the formula of De Moivre.

Equation (1) gives

$$d\varphi = \frac{-d \cos \varphi}{(1 - \cos^2 \varphi)^{\frac{1}{2}}}.$$

Multiplying both sides by $\sqrt{-1}$, we shall have

$$(7) \qquad \sqrt{(-1)}\, d\varphi = \frac{-d\cos\varphi}{(\cos^2\varphi - 1)^{\frac{1}{2}}},$$

the integral of which is

$$(8) \qquad \sqrt{(-1)}\, \varphi = \log(\cos\varphi + \sqrt{(-1)} \sin\varphi).$$

No constant is required, for when $\varphi = 0$ then $\sin\varphi = 0$, and $\cos\varphi = 1$, whose $\log = 0$. Changing (8) to its exponential form, we get

$$(9) \qquad \cos\varphi + \sqrt{(-1)} \sin\varphi = e^{\varphi\sqrt{-1}},$$

which is the formula of EULER.

Equation (6) can be easily derived from (9); if we put $n\varphi$ for φ, we shall get

$$(10) \qquad \cos n\varphi + \sqrt{(-1)} \sin n\varphi = e^{n\varphi\sqrt{-1}};$$

then if we raise both sides of (9) to the nth power, we shall have

$$(11) \qquad (\cos\varphi + \sqrt{(-1)} \sin\varphi)^n = e^{n\varphi\sqrt{-1}};$$

hence from (10) and (11) we get

$$(\cos\varphi + \sqrt{(-1)} \sin\varphi)^n = \cos n\varphi + \sqrt{(-1)} \sin n\varphi,$$

which is the same as equation (6), or DE MOIVRE'S formula deduced from EULER'S.

Again, if in equation (9) we put $\varphi = a$, b, and $(a + b)$, successively, we shall get

$$(12) \qquad \cos a + \sqrt{(-1)} \sin a = e^{a\sqrt{-1}},$$

$$(13) \qquad \cos b + \sqrt{(-1)} \sin b = e^{b\sqrt{-1}},$$

$$(14) \qquad \cos(a + b) + \sqrt{(-1)} \sin(a + b) = e^{(a+b)\sqrt{-1}}.$$

Multiplying (12) and (13) together and comparing the product with (14), equating irrational parts with irrational parts, and rational parts with rational parts, we shall get

(15) $$\sin(a + b) = \sin a \cos b + \cos a \sin b,$$

(16) $$\cos(a + b) = \cos a \cos b - \sin a \sin b,$$

which are fundamental equations in trigonometry.

Also, if in (6) we put $n = 2$, we shall get

$$(\cos\varphi + \sqrt{(-1)}\sin\varphi)^2 = \cos 2\varphi + \sqrt{(-1)}\sin 2\varphi;$$

expanding the binomial, and equating irrational parts, we shall get

$$\sin 2\varphi = 2\sin\varphi\cos\varphi,$$

$$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi.$$

It is evident that (15) and (16) will give the same results if we put $a = b = \varphi$. Again, if in equation (9) we write $-\varphi$ for φ, we shall have

(17) $$\cos\varphi - \sqrt{(-1)}\sin\varphi = e^{-\varphi\sqrt{-1}};$$

from (9) and (17) we shall obviously get

$$\sin\varphi = \frac{e^{\varphi\sqrt{-1}} - e^{-\varphi\sqrt{-1}}}{2\sqrt{-1}},$$

$$\cos\varphi = \frac{e^{\varphi\sqrt{-1}} + e^{-\varphi\sqrt{-1}}}{2}.$$

If we develop the exponentials in these equations, we shall get

$$\sin\varphi = \varphi - \frac{\varphi^3}{1.2.3} + \frac{\varphi^5}{1.2.3.4.5} - \&c.,$$

$$\cos\varphi = 1 - \frac{\varphi^2}{1.2} + \frac{\varphi^4}{1.2.3.4} - \frac{\varphi^6}{1.2.3.4.5.6} + \&c.$$

Again, if in equation (8) we put $\varphi = 180° = \pi$, we shall get

$$\sqrt{(-1)}\,\pi = \log(-1);$$

therefore

$$\pi = \frac{\log(-1)}{\sqrt{-1}}.$$

For other applications of these equations see CHAUVENET's Trigonometry, and MOIGNO's Differential Calculus.

ON THE INDETERMINATE ANALYSIS.

By Rev. A. D. Wheeler, Brunswick, Maine.

[Continued from Page 57.]

Proposition X. The least value of c for n solutions, in the equation $ax+by=c$, is $c=(n-1)ab+a+b$.

Demonstration. Let $x=1$ and $y=1$; since these are the least values that can be given them. Then will the succeeding values of x be $1+b$, $1+2b$, &c., to $1+(n-1)b$, for the nth solution. (Prop. VII.) Substituting for x and y their least values for the nth solution, as thus obtained, we have

$$ax+by=a(1+(n-1)b)+b=(n-1)ab+a+b=c;$$

which is the value required.

Prop. XI. When $c=nab$, the equation $ax+by=c$ admits of only $n-1$ solutions, in whole numbers.

Dem. Put v for the first value of x. Then will $v+(n-1)b$ be the nth value as before.

By substitution, $ax+by=av+(n-1)ab+by=nab$, according to the supposition. Reducing, we obtain $av+by=ab$, which admits of no solution; as shown in Prop. IX. Case 3. All the intermediate values of x will render the equation possible. Therefore the whole number of solutions is $n-1$.

Prop. XII. The greatest value of c for n solutions is $c=(n+1)ab$.

Dem. Let $x=v$ be the first value of x, and $x=v+(n-1)b$ be the nth, as in the preceding proposition. Then

$$ax+by=av+(n-1)ab+by=(n+1)ab,$$

according to the supposition. Reducing, we obtain the equation $av+by=2ab$. This is possible for one solution, as shown in Prop. VIII., and for only one, as shown in Prop. XI. Therefore the equation $ax+by=(n+1)ab$ is possible for n solutions and no

more. Let $n + 1 = n'$. Now if c were greater than $n'ab$; that is, if $c = n'ab + r$; then, by Prop. VIII., the equation would admit of n' or $n + 1$ solutions. Therefore $(n + 1)\,ab$ is the greatest value of c for n solutions.

Prop. XIII. When $c = nab + ax' + by'$, the equation $ax + by = c$ is always possible for $n + 1$ solutions.

Dem. We have $ax + by = nab + ax' + by'$. Let $x = v$, for its first value. Then for the $(n + 1)$th value we shall have $v + nb$, (Prop. VII.), and the equation becomes

$$av + nab + by = nab + ax' + by', \text{ or}$$
$$av + by = ax' + by';$$

this is clearly possible, since we can always make $v = x'$ and $y = y'$.

Remark. — If we divide $nab + ax' + by'$ by ab, we obtain the quotient n, and the remainder $ax' + by'$. Now the equation $av + by = ax' + by'$, will always admit of one solution, for the reason which has been given. It cannot admit of more than one, since $ax' + by' > ab$; and the least value for two solutions, according to Prop. X., is $c = ab + a + b$. Further, as it has been proved (Prop. XI.) that $c = nab$ will give $n - 1$ solutions; and (Prop. VIII.) that $c > nab$ will give n solutions; we are able to determine, in all cases, the exact number of solutions which any equation of this form admits of, by the following simple

Rule. — Divide c by ab. If there be no remainder, the whole number of solutions will be one less than the quotient. If there be a remainder and that remainder admits of a solution, it will be one more than the quotient. In all other cases it will be equal to the quotient.

Prop. XIV. If the numbers 1, 2, 3, &c., in their order, be substituted for c, until we arrive at the value $c = ab$, the possible equations, of the form $ax + by = ax' + by'$, in this series, will always be equal to the impossible equations, of the form $ax + by = ab - (ax' + by')$.

DEM. Let $ax + by = ab$,

and $ax'' + by'' = ab - (ax' + by')$, an impossible equation. Subtracting, $a(x - x'') + b(y - y'') = ax' + by'$; which is possible, since we can make $x - x'' = x'$, and $y - y'' = y'$. Thus there is a possible equation for every impossible one of this form, and, of course, the number of each must be equal.

[To be Continued.]

THE ELEMENTS OF QUATERNIONS.

[Continued from Page 175.]

VI. GENERAL FORMULÆ.

39. Two special cases of equations (39′) deserve notice. When q is a tensor or a scalar, they become

(49) $\quad \mathrm{S}qr = q\,\mathrm{S}r, \qquad$ and $\quad \mathrm{V}qr = q\,\mathrm{V}r.$

And when $r = q$, they become, by (17),

(50) $\quad \mathrm{S}q^2 = (\mathrm{S}q)^2 + (\mathrm{V}q)^2, \qquad$ and $\mathrm{V}q^2 = 2\,\mathrm{S}q\,.\,\mathrm{V}q.$

The sum and difference of (50) and (35) give

(51) $\quad (\mathrm{S} + \mathrm{T})\,q^2 = 2\,(\mathrm{S}q)^2, \qquad (\mathrm{S} - \mathrm{T})\,q^2 = 2\,(\mathrm{V}q)^2.$

40. Since, by (24), $\mathrm{V}r\,.\,\mathrm{V}q = \mathrm{K}(\mathrm{V}q\,.\,\mathrm{V}r)$, (39′) gives by (20),

$$(52)\quad \begin{aligned} \mathrm{S}rq &= \mathrm{S}r.\mathrm{S}q + \mathrm{S}(\mathrm{V}r.\mathrm{V}q) & \mathrm{V}rq &= \mathrm{S}r.\mathrm{V}q + \mathrm{S}q.\mathrm{V}r + \mathrm{V}(\mathrm{V}r.\mathrm{V}q) \\ &= \mathrm{S}r.\mathrm{S}q + \mathrm{S}(\mathrm{V}q.\mathrm{V}r), & &= \mathrm{S}r.\mathrm{V}q + \mathrm{S}q.\mathrm{V}r - \mathrm{V}(\mathrm{V}q.\mathrm{V}r). \end{aligned}$$

Equations (39′) and (52) give

(53) $\quad \mathrm{S}qr = \mathrm{S}rq,$

(54) $\quad \mathrm{V}qr + \mathrm{V}rq = 2(\mathrm{S}r.\mathrm{V}q + \mathrm{S}q.\mathrm{V}r), \quad \mathrm{V}qr - \mathrm{V}rq = 2\mathrm{V}(\mathrm{V}q.\mathrm{V}r).$

41. By the aid of the associative principle, we may extend (8) and (15) to

(55) $\quad \mathrm{K}\Pi = \Pi'\mathrm{K}, \qquad$ and $(\Pi q)^{-1} = \Pi' q^{-1},$

in which Π' is used to denote a product composed of the same fac-

tors as Π, but taken in the reverse order. Equations (21) give by (55)

$$(56)\qquad 2\,\mathrm{S}\,\Pi = (1+\mathrm{K})\,\Pi = \Pi + \Pi'\mathrm{K},\qquad 2\,\mathrm{V}\,\Pi = \Pi - \Pi'\mathrm{K}.$$

From equations (55) and (56) may be deduced, by means of (20), for the *products of vectors*,

$$(57)\qquad \mathrm{K}\,\Pi = \pm\,\Pi',$$

$$(58)\qquad \mathrm{S}\,\Pi = \tfrac{1}{2}(\Pi \pm \Pi') = \pm\,\mathrm{S}\,\Pi',\quad \mathrm{V}\,\Pi = \tfrac{1}{2}(\Pi \mp \Pi') = \mp\,\mathrm{V}\,\Pi',$$

the upper signs being used when the number of factors in Π is even, and the under ones when it is odd.

Equations (39′) may, by means of (19 a) and (24), be thus deduced as a special case of (56),

$$\begin{aligned}2\,\mathrm{S}\,qr &= qr + \mathrm{K}r.\mathrm{K}q = (\mathrm{S}q+\mathrm{V}q)(\mathrm{S}r+\mathrm{V}r)+(\mathrm{S}r-\mathrm{V}r)(\mathrm{S}q-\mathrm{V}q)\\ &= 2\mathrm{S}q.\mathrm{S}r + \mathrm{V}q.\mathrm{V}r + \mathrm{V}r.\mathrm{V}q = 2\mathrm{S}q.\mathrm{S}r + (1+\mathrm{K})(\mathrm{V}q.\mathrm{V}r)\\ &= 2\,\mathrm{S}q.\mathrm{S}r + 2\,\mathrm{S}(\mathrm{V}q.\mathrm{V}r);\end{aligned}$$

and similarly

$$2\,\mathrm{V}\,qr = qr - \mathrm{K}r.\mathrm{K}q = 2\,\mathrm{S}q.\mathrm{V}r + 2\,\mathrm{S}r.\mathrm{V}q + 2\,\mathrm{V}(\mathrm{V}q.\mathrm{V}r).$$

42. By means of (36) and (34′), equation (53) may be transformed to

$$\mathrm{T}q.\mathrm{T}r.\cos\angle qr = \mathrm{T}q.\mathrm{T}r.\cos\angle rq;\ \text{whence}$$

$$(59)\qquad \angle qr = \pm\,\angle rq.$$

Whence it follows that $\sin\angle qr = \pm\sin\angle rq$, and therefore, by (34″),

$$(60)\qquad \mathrm{T\,V\,U}\,qr = \mathrm{T\,V\,U}\,rq.$$

Using P to denote a product composed of the same factors as Π, but altered in their arrangement by any cyclical permutation, equations (53), (59), and (60) may be generalized as follows:

$$(61)\qquad \mathrm{S}\,\Pi = \mathrm{S\,P},\quad \angle\Pi = \pm\,\angle\mathrm{P},\quad \mathrm{T\,V\,U}\,\Pi = \mathrm{T\,V\,U\,P}.$$

Taking the tensor of (36), we find $\mathrm{T\,V}\,q = \mathrm{T}q.\mathrm{T\,V\,U}\,q$; whence

$$(62)\qquad \mathrm{T\,V}\,\Pi = \mathrm{T\,V\,P}.$$

NOTE. — The following simple applications of quaternion analysis are appended by way of examples.

I. Let the vectors α, β, γ form the three sides of a triangle, their positive directions being indicated by the arrows in the figure.

By § 3, $\qquad \gamma = \alpha - \beta;$

therefore $\qquad \gamma^2 = \alpha^2 - \alpha\beta - \beta\alpha + \beta^2,$

by (17) and (24), $\qquad = - \mathrm{T}\,\gamma^2 = - \mathrm{T}\,\alpha^2 - (1 + \mathrm{K})\,\alpha\beta - \mathrm{T}\,\beta^2;$

by (21), $\qquad \mathrm{T}\,\gamma^2 = \mathrm{T}\,\alpha^2 + 2\,\mathrm{S}\,\alpha\beta + \mathrm{T}\,\beta^2;$

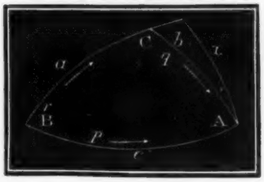
by (31) and (33),

$$= \mathrm{T}\,\alpha^2 + 2\,\mathrm{T}\,\alpha\beta \,.\, \cos\frac{\alpha}{\beta^{-1}} + \mathrm{T}\,\beta^2$$

$$= \mathrm{T}\,\alpha^2 + \mathrm{T}\,\beta^2 - 2\,\mathrm{T}\,\alpha \,.\, \mathrm{T}\,\beta \,.\, \cos\frac{\alpha}{\beta};$$

but, as $\mathrm{T}\,\alpha$, &c., represent the lengths of the sides of the triangle, this equation expresses the well-known relation $c^2 = a^2 + b^2 - 2\,a\,b\cos\mathrm{C}$.

II. Let the *versor quaternions* p, q, r form the three sides of a spherical triangle, (*see note to* § 18), the positive directions of their angles being indicated by the arrows in the figure. If the usual notation be also employed for the sides and angles, we shall have



$$\mathrm{C} = \frac{\mathrm{Ax.}\,r^{-1}}{\mathrm{Ax.}\,q}, \qquad a = \angle\, r, \qquad b = \angle\, q, \qquad c = \angle\, p.$$

A, B, and C, may also be employed to denote the directions from the centre of the sphere to the respective vertices of these angles.

By § 9, $\qquad p = q\,r;$

by (39′), $\qquad \mathrm{S}\,p = \mathrm{S}\,q \,.\, \mathrm{S}\,r + \mathrm{S}\,(\mathrm{V}\,q \,.\, \mathrm{V}\,r),$

by (31) and (33), $\qquad = \mathrm{S}\,q \,.\, \mathrm{S}\,r + \mathrm{T}\,(\mathrm{V}\,q \,.\, \mathrm{V}\,r)\cos\frac{\mathrm{Ax.}\,q}{\mathrm{Ax.}\,r^{-1}};$

similarly, $\qquad \mathrm{V}\,p = \mathrm{S}\,q \,.\, \mathrm{V}\,r + \mathrm{S}\,r \,.\, \mathrm{V}\,q + \mathrm{V}\,(\mathrm{V}\,q \,.\, \mathrm{V}\,r),$

by (33 a), $\qquad = \mathrm{S}\,q \,.\, \mathrm{V}\,r + \mathrm{S}\,r \,.\, \mathrm{V}\,q + \mathrm{T}\,(\mathrm{V}\,q \,.\, \mathrm{V}\,r)\sin\frac{\mathrm{Ax.}\,q}{\mathrm{Ax.}\,r^{-1}} \,.\, \sqrt{-1}_{\mathrm{C}}.$

By (34′) and (34″), the first of these two equations is equivalent to the well-known *fundamental equation* of spherical trigonometry,

$$\cos c = \cos a\cos b + \sin a\sin b\cos\mathrm{C};$$

and the second is equivalent to

$$(m)\quad \sin c \,.\, \sqrt{-1}_p = \cos b\sin a \,.\, \sqrt{-1}_r + \cos a\sin b \,.\, \sqrt{-1}_q - \sin a\sin b\sin\mathrm{C} \,.\, \sqrt{-1}_{\mathrm{C}}.$$

This last equation, being the value of a vector, involves, by § 27, three independent elements, and is therefore equivalent to three *independent* equations. These might be obtained by projection in any three mutually rectangular directions, as in § 27. We

will develop the projections in the directions of Ax. q and A; and for this purpose we must multiply each term of (m) by the cosine of the angle between the vector involved in that term and the direction of Ax. q or A. The angle between the direction A and the plane of r being denoted by x, the multipliers of these terms are respectively

for the direction Ax. q, the cosines of the angles A, $180° - C$, $0°$, $90°$;
" " A " " " $90°$, $90° - x$, $90°$, b.

The two equations thus derived from (m) are therefore

$$\sin c \cos A = - \cos b \sin a \cos C + \cos a \sin b,$$
$$0 = \cos b \sin a \sin x - \sin a \sin b \sin C \cos b.$$

The first of these is another *fundamental equation*, and the second gives, by dividing by $\sin a \cos b$, simply the equation

$$\sin x = \sin b \sin C.$$

PROPERTIES OF CURVATURE IN THE ELLIPSE AND HYPERBOLA.

By Chauncey Wright, Nautical Almanac Office, Cambridge, Mass.

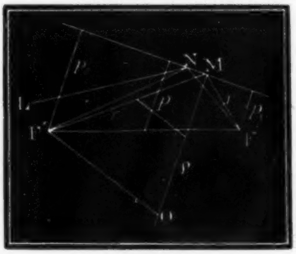
The equation of curvature, $\varrho = \frac{r\,r'}{p}$, (which, as furnishing a direct and simple means of constructing points in the evolute of the ellipse or hyperbola by the method of the fourth proportional, was published in No. X. Vol. I. of this Journal, among the Prize Problems,) can itself be easily deduced from a simple geometrical construction.

1. In this equation r and r' denote the radii vectores from the foci to any point of the curve; p denotes the perpendicular line from the centre to the tangent of this point, and ϱ the radius of curvature.

Further, let p_1 and p_2 denote the perpendicular lines from the foci to the tangent, and let ε denote the angle formed by both the radii with the tangent; A and B the semi-axes of the curve, and φ the angle formed by the radius r with the axis A.

If to two points, M and N, of an ellipse or an hyperbola, distant from each other by the element of arc $d\,s$, radii be drawn from both foci, and if the angle included by the radii from the first focus F be

denoted by $d\varphi$, and if a tangent be drawn through one of these points M, and the radius from the first focus F to the other point N be extended to this tangent (an infinitesimal distance of the second order), and from the point of intersection the line NL be drawn, forming the same angle with the tangent as the intersecting radius; then the angle included between this line NL and the radius r' from the second focus F' to the first point M, will be $d\varphi$, and the distance apart of these two lines at the second focus F' will be in the ellipse, $(r + r')\, d\varphi$, and in the hyperbola $(r' - r)\, d\varphi$, or in both $2\, A\, d\varphi$.



ELLIPSE.

HYPERBOLA.

The angle which the line NL forms with the other radius NF' from the second focus is obviously equal to twice the angle which the tangents of the two points M and N form with each other, and we may express it by $2\, d\tau$. The distance apart of these lines at the second focus is therefore $2\, r'\, d\tau$; hence the distances $2\, A\, d\varphi$ and $2\, r'\, d\tau$ are equal or

$$A\, d\varphi = r'\, d\tau;$$

but

$$r\, d\varphi = d s \sin \varepsilon,$$

and dividing the latter equation by the former we have

$$\frac{r}{A} = \frac{d s \sin \varepsilon}{r'\, d\tau}, \text{ hence } \frac{r r'}{A \sin \varepsilon} = \frac{d s}{d \tau} = \varrho, \text{ the radius of curvature.}$$

Since from the figures $p_1 = r \sin \varepsilon$ and $p_2 = r' \sin \varepsilon$; p, which is the arithmetical mean of p_1 and p_2, is in the ellipse equal to $\frac{r' + r}{2} \sin \varepsilon$, and in the hyperbola $\frac{r' - r}{2} \sin \varepsilon$, or in both

$$p = A \sin \varepsilon;$$

hence $$\varrho = \frac{r\, r'}{p}.$$

The construction of the centre of curvature O is given in the figures.

2. The equation of curvature $\varrho = \frac{A^2 B^2}{p^3}$ may be deduced from the one above by the theorem,

$$p_1 p_2 = B^2,$$

which is easily proved as follows.

The angle included by the radii vectores to any point is in the ellipse the supplement of $2\,\varepsilon$ and in the hyperbola it is equal to $2\,\varepsilon$. If now the distance between the foci be denoted by $2\,C$ we have by trigonometry

$$4\,C^2 = r^2 \pm 2\,r\,r' \cos 2\,\varepsilon + r'^2,$$

(the upper sign for the ellipse and the lower sign for the hyperbola) and since $\cos 2\,\varepsilon = 1 - 2 \sin^2 \varepsilon$ we have

$$4\,C^2 = r^2 \pm 2\,r r' + r'^2 \mp 4\,r r' \sin^2 \varepsilon = (r \pm r')^2 \mp 4\,r r' \sin^2 \varepsilon = 4\,A^2 \mp 4\,r r' \sin^2 \varepsilon;$$

hence $$r\,r' \sin^2 \varepsilon = \pm (A^2 - C^2) = B^2 = p_1 p_2.$$

$$\therefore r\,r' = \frac{B^2}{\sin^2 \varepsilon} \text{ and } \frac{r\,r'}{A \sin \varepsilon} = \frac{B^2}{A \sin^3 \varepsilon} = \frac{A^2 B^2}{p^3} = \varrho.$$

3. The mechanical properties of the ellipse and hyperbola may be easily deduced from the equations

$$\varrho = \frac{A^2 B^2}{p^3} = \frac{B^2}{A \sin^3 \varepsilon} = \frac{r\,r'}{p} = \frac{r\,r'}{A \sin \varepsilon},$$

in the two cases of central forces for which these curves are the paths described.

First, if the centre of attraction or repulsion be the centre of the ellipse or hyperbola, and if v denote the velocity with which the material point is at any time, t, moving, and r_c the radius vector of this point from the centre, ε_c the angle which it forms with the tan-

gent and φ_c the angle which it forms with the axis A; the principle of *equal areas* may be expressed by the equation,

$$p\,v = a = \text{a constant},$$

or putting $p = r_c \sin \varepsilon_c$ and $v = \frac{ds}{dt}$, and if $d\sigma =$ the elementary area described by the radius r_c in the time dt we have

$$r_c \sin \varepsilon_c \times \frac{ds}{dt} = a = \frac{r_c^2\, d\varphi_c}{dt} = \frac{2\, d\sigma}{dt}.$$

If the area and time be reckoned from the same origin, and if T denote the time of one complete revolution, a is equal to twice the whole area of the ellipse divided by T or $a = \frac{2AB\pi}{T}$.

If we substitute in the equation of *living force*

$$d(v^2) = 2\,R\,d\,r_c$$

(in which R is the force of attraction or repulsion) the value of v from the equation of equal areas $v = \frac{a}{p}$ we get

$$d\left(\frac{a^2}{p^2}\right) = -\frac{2\,a^2}{p^3}\,dp = 2\,R\,d\,r_c,$$

and hence $$R\frac{d\,r_c}{dp} = -\frac{a^2}{p^3}.$$

But in general $$\varrho = r_c\frac{d\,r_c}{dp};*$$

hence in the ellipse and hyperbola

$$r_c\frac{d\,r_c}{dp} = \frac{A^2B^2}{p^3} = -\frac{a^2}{Rp^3}r_c;$$

hence $$R = -\frac{a^2}{A^2B^2}r_c,$$

or the force is proportional to the distance from the centre.

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* It is readily seen by the construction of these quantities that $dp_x = r_x \cos \varepsilon_x\, d\tau$, $dr_x = ds \cos \varepsilon_x$, and hence the ratio $\frac{d\,r_x}{dp_x} = \frac{ds}{r_x\, d\tau} = \frac{\varrho}{r_x}$; in which p_x, r_x, and ε_x, are the perpendicular to the tangent, the radius vector and its angle with the tangent, for any centre and any curve whatever.

Since in the ellipse $a = \frac{2\,A\,B\,\pi}{T}$ we get $R = -\frac{4\,A^2\,B^2\,\pi^2}{A^2\,B^2\,T^2}\,r_c = -\frac{4\,\pi^2}{T^2}\,r_c$; hence the time of revolution is independent of the magnitude or form of the ellipse. The small oscillations of a free pendulum are therefore isochronous.

Secondly, when one of the foci F of the ellipse or hyperbola is the centre of force, we have the mechanical equations

$$v = \frac{a}{p_1} \text{ and } d(v^2) = 2\,R\,d\,r = -\frac{2\,a^2}{p_1^{\,3}}\,d\,p_1,$$

and the equations of curvature

$$\varrho = r\,\frac{d\,r}{d\,p_1} = \frac{B^2}{A\,\sin^3\varepsilon};$$

hence $R\,\frac{d\,r}{d\,p_1} = -\frac{a^2}{p_1^{\,3}} = -\frac{a^2}{r^3\sin^3\varepsilon} = R\,\frac{B^2}{A\,r\,\sin^3\varepsilon}$ and $R = -\frac{A\,a^2}{B^2\,r^2}$;

the force is therefore inversely proportional to the square of the distance, and in the ellipse, since $a = \frac{2\,A\,B\,\pi}{T}$,

$$R = -\frac{4\,A^3\,B^2\,\pi^2}{T^2\,B^2\,r^2} = -\frac{4\,A^3\,\pi^2}{T^2\,r^2}.$$

The time of revolution is therefore independent of the minor axis B, and its square is proportional to the third power of the major axis, according to KEPLER's third law.

The general equation

$$R = -\frac{a^2\,A\,r_x\,\sin^3\varepsilon}{B^2\,p_x^{\,3}}$$

includes the two cases above, and all cases in which r_x and p_x are the radius and perpendicular to the tangent from any centre of force whatever.

If we substitute in it the radius and perpendicular from the centre, r_c and $p = A\,\sin\varepsilon$, or the radius and perpendicular from the

focus, r and $p_1 = r \sin \varepsilon$, the force R becomes independent of direction, and is a function of the radius only.

If we resume the mechanical equations

$$v = \frac{a}{p_x} \text{ and } d(v^2) = 2\, R\, d\, r_x = \frac{2\, a^2}{p_x^3}\, d\, p_x,$$

and the equation of curvature

$$\frac{\varrho}{r_x} = \frac{d\, r_x}{d\, p_x},$$

we have in general

$$\frac{R\, \varrho}{r_x} = -\frac{a^2}{p_x^3} = -\frac{v^2}{p_x};$$

hence

$$R \frac{p_x}{r_x} = -\frac{v^2}{\varrho}.$$

But $R \frac{p_x}{r_x} = R \sin \varepsilon_x$ is the component of R perpendicular to the path of the moving point, and is equal and opposite to the centrifugal force of this point. Hence this centrifugal or normal force $N = -R \sin \varepsilon_x = \frac{v^2}{\varrho}$. When the centre of force is the centre of the ellipse or hyperbola, $N = \frac{v^2}{\varrho} = m\, r_c \sin \varepsilon_c = m\, p$ (m being the force at the distance unity from this centre). But from the equation of curvature $\varrho = \frac{r\, r'}{p}$ we have $v^2 = m\, p\, \varrho = m\, r\, r'$.

When the centre of force is the focus F of the ellipse or hyperbola we have $N = \frac{v^2}{\varrho} = \frac{m \sin \varepsilon}{r^2}$, and from the equation of curvature $\varrho = \frac{r\, r'}{A \sin \varepsilon}$ we have $v^2 = \varrho\, \frac{m \sin \varepsilon}{r^2} = \frac{m\, r'}{A\, r}$.

The square of the velocity is therefore proportional in the first case to the product of the radii from the foci; and in the second case to their ratio. Moreover, it is obvious that in the second case the product of any two velocities in two positions at equal and opposite distances from the minor axis is the constant $\frac{m}{A}$; that is,

one velocity is as many times greater than the constant $\sqrt{\frac{m}{A}}$, as the other is less.

Since we have by geometry $r^2 + r'^2 = 2 r_c^2 + 2 C^2$, we find

$$4 A^2 = (r + r')^2 = r^2 + 2 r r' + r'^2 = 2 r_c^2 + 2 C^2 + 2 r r';$$

hence $$r r' = 2 A^2 - C^2 - r_c^2 = A^2 + B^2 - r_c^2,$$

and $$v^2 = m (A^2 + B^2 - r_c^2).$$

In the second case we have

$$v^2 = \frac{m r'}{A r} = m \frac{2 A - r}{A r} = m \left(\frac{2}{r} - \frac{1}{A}\right).$$

These values of v^2 may also be deduced, though not so readily, by integration, and the determination of constants from the fundamental equation $d(v^2) = 2 R\, d r_x$.

LAW OF GRAVITY.

Problem in Celestial Mechanics.

Prove that the Newtonian law of gravity is such, that the scale of spaces and the scale of times, in the motions of the heavenly bodies, are independent of each other; that is, prove that in two systems similar in construction, but different in scale, all the corresponding motions will be synchronous, whether in the bodies themselves, or in bodies on their surfaces.

PROFESSOR ENCKE'S METHOD OF COMPUTING SPECIAL PERTURBATIONS.*

After the choice of the magnitude of the intervals and of the time of the beginning is settled, either by general considerations, or by a preliminary calculation, the computation arranges itself into the five following divisions: —

1. The computation of the places of the disturbing planet for the assumed times, and their reduction to the values L', r', ☊$'$, i', adopted in the formulæ.

2. The computation of the places of the disturbed planet, and of the values necessary to form the coefficients in the differential equations.

3. The computation of the amount of the disturbing forces in the directions R, S, W.

4. Their substitution in the equations of condition, and the calculation of the values of the differential coefficients themselves.

5. The integration of the latter, or the construction of the table, which exhibits the values of the general integral combined with the different constants.

Our planetary tables give directly the longitude in the orbit, and the radius vector, and therefore the values denoted by L', r', ☊$'$, and i'. So that if the tables are immediately resorted to, by adhering to these values, the perturbations in latitude only will be neglected, which are always so small that their entire omission is of no importance. In this case, it will be necessary to reduce the lon-

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* Jahrbuch, 1838. Translated from the German for the Use of the American Ephemeris and Nautical Almanac, by Charles Henry Davis, Commander United States Navy, Superintendent of the Nautical Almanac.

gitudes collectively, by the addition of the precession, to a fixed mean equinox, usually that of the time of the beginning.

If, on the contrary, the places are interpolated from the ephemeris which give the longitudes in the ecliptic, and the latitudes, it will be necessary to make use of the latter with rigorous exactness in order to find Ω' and i', and thence to deduce the values of L', the longitudes in the orbit. If l' and b' are the longitudes and latitudes already reduced to the fixed mean equinox, then the nutation with the opposite sign and the precession are already applied, and it will be necessary to compute from two values l_0' and l_1', b_0' and b_1', selected as advantageously as the circumstances will allow, the following formulæ: —

$$\sin\left(\tfrac{1}{2}(l_1' + l_0') - \Omega'\right)\tan i' = \frac{\sin(b_1' + b_0')}{2\cos b_1' \cos b_0' \cos\frac{1}{2}(l_1' - l_0')};$$

$$\cos\left(\tfrac{1}{2}(l_1' + l_0') - \Omega'\right)\tan i' = \frac{\sin(b_1' - b_0')}{2\cos b_1' \cos b_0' \sin\frac{1}{2}(l_1' - l_0')};$$

$$L' = l' + \tan\tfrac{1}{2}i'^2 \sin 2(l' - \Omega') + \tfrac{1}{2}\tan\tfrac{1}{2}i'^4 \sin 4(l' - \Omega') \ldots .$$

the last formula for every l'; from the first two are obtained Ω' and i', including the perturbations in latitude as accurately as the tables permit. For all the older planets the term containing $\tan\frac{1}{2}i'^4$ is nearly insensible. In this manner the first part of the computation is given in the desired form.

For the second, or the place of the disturbed planet, it will be convenient to separate, in the expression of the differential coefficients, the quantities which, depending merely upon the elements remain constant during the greater number of intervals, from the other quantities varying with the time. If we designate by R_0, S_0, W_0, for the present, what has been denoted in the equations (23) by $\frac{kR_0}{\sqrt{p}}$, $\frac{kS_0}{\sqrt{p}}$, $\frac{kW_0}{\sqrt{p}}$, and if we call the amount of one interval expressed in mean days ω, observing, moreover, that for the purpose of integration the differential coefficients must be multiplied by ω, with the

exception of $\frac{d\mu}{dt}$, which on account of the double integral must be multiplied by ω^2, and if we introduce for the constant factors the notation (1), (2), (3), &c., we shall have the following formulæ: —

$(1) = \frac{k\,m'}{\sqrt{p}}\,\omega$, where m' is expressed in seconds, or $\log m' = 5.3144251$ + Brigg's log of the mass of the disturbing planet in parts of the sun's mass, and $\log k = 8.2355814$;

$(2) = \frac{1}{\sin i}$; $(3) = a \cos \varphi$; $(4) = \frac{p}{e}$;

$(5) = \frac{1}{e}$; $(6) = \tan \frac{1}{2} i$; $(7) = \frac{3\,k\,\omega}{\sqrt{a}}\, e$;

$(8) = \frac{3\,k\,\omega}{\sqrt{a}}\, p$; $(9) = p \tan \frac{1}{2} \varphi$; $(10) = 2 \cos \varphi$;

$(11) = \tan \frac{1}{2} \varphi$; $(12) = p \cot \varphi$; $(13) = \cot \varphi$.

The equations (22) and (23) are

$$(22) \qquad R_0 = (1)\ R'; \qquad S_0 = (1)\ S'; \qquad W_0 = (1)\ W';$$

$$\omega^2 \frac{d\mu}{dt} = -\frac{3k}{\sqrt{a}}\, e \sin v\ R_0 - \frac{3k}{\sqrt{a}} \cdot \frac{p}{r} \cdot S_0;$$

$$\omega \frac{dM}{dt} = (2\,r \cos \varphi - p \cot \varphi \cos v)\ R_0$$
$$- (p + r) \cot \varphi \sin v\ S_0 + \omega \int \frac{d\mu}{dt} \cdot dt;$$

$$(23) \qquad \omega \frac{d\varphi}{dt} = a \cos \varphi \sin v\ R_0 + a \cot \varphi \left(\frac{p}{r} - \frac{r}{a}\right) S_0;$$

$$\omega \frac{d\pi}{dt} = -\frac{p \cos v}{e} \cdot R_0 + \frac{p + r}{e} \sin v\ S_0 + (1 - \cos i)\ \omega \frac{d\Omega}{dt};$$

$$\omega \frac{d\Omega}{dt} = \frac{r \sin (v + \pi - \Omega)}{\sin i} \cdot W_0;$$

$$\omega \frac{di}{dt} = r \cos (v + \pi - \Omega) \cdot W_0.$$

Or, if we introduce L in the place of M,

$$\omega \frac{dL}{dt} = -(2\,r \cos \varphi + p \tan \tfrac{1}{2} \varphi \cos v)\ R_0$$
$$+ (p + r) \tan \tfrac{1}{2} \varphi \sin v\ S_0 + (1 - \cos i)\ \omega \frac{d\Omega}{dt} + \omega \int \frac{d\mu}{dt} \cdot dt.$$

With the above factors, the equations (23) will be written as follows: —

$$\omega \frac{di}{dt} = r \cos u \, W_0; \qquad \cos u = \cos (v + \pi - \Omega).$$

$$\omega \frac{d\Omega}{dt} = (2)\, r \sin u \, W_0;$$

$$\omega \frac{d\varphi}{dt} = (3) \sin v \, R_0 + (3)(\cos v + \cos E)\, S_0;$$

$$\omega \frac{d\pi}{dt} = -(4) \cos v \, R_0 + (5)\left(\frac{p}{r} + 1\right) r \sin v \, S_0 + (6)\, r \sin u \, W_0;$$

$$\omega^2 \frac{d\mu}{dt} = -(7) \sin v \, R_0 - (8) \frac{1}{r} S_0;$$

$$\omega \frac{dL}{dt} = \{-(9) \cos v - (10)\, r\}\, R_0 + (11)\left(\frac{p}{r} + 1\right) r \sin v \, S_0 + (6)\, r \sin u \, W_0 + \omega \int \frac{d\mu}{dt}\, dt;$$

$$\omega \frac{dM}{dt} = \{+(12) \cos v - (10)\, r\}\, R_0 - (13)\left(\frac{p}{r} + 1\right) r \sin v \, S_0 + \omega \int \frac{d\mu}{dt}\, dt.$$

These formulæ are wholly identical with (23), except this small change, that, since

$$\frac{p}{r} = 1 + e \cos v, \qquad \frac{r}{a} = 1 - e \cos E,$$

$e(\cos v + \cos E)$ is written instead of $\frac{p}{r} - \frac{r}{a}$, in consequence of which the last term in $\frac{d\varphi}{dt}$ is changed to

$$a \cos \varphi (\cos v + \cos E).$$

It is preferable in numerical computation to add together two small quantities which have with few exceptions the same signs, rather than to subtract from each other two larger quantities of

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NOTE. — The equations (22) and (23) are the final equations for the perturbations of the elements in the theoretical investigation.

which the sign is always the same. This addition will be particularly facilitated by the Gaussian table of logarithms of sums and differences, which table can be frequently used with great advantage in this calculation. On account of using this table, $\left(\frac{p}{r}+1\right) r \sin v$ is written instead of $(p+r)\sin v$. If $p > r$, and $\frac{p}{r}$ therefore an improper fraction, for A in the Gaussian table taken as the $\log\frac{p}{r}$ the adjoining logarithm in the column C is equal to $\log\left(\frac{p}{r}+1\right)$; and again, if $p < r$, or $\frac{p}{r}$ is a proper fraction, the column B gives the $\log\left(\frac{p}{r}+1\right)$ for $A = \log\frac{r}{p}$; thus we enter always A with the difference of the logarithms of p and r, and find then either in B or C the $\log\left(\frac{p}{r}+1\right)$ according as the latter is smaller or greater than the logarithm of the number two, which can be immediately seen.

We have, therefore, upon the whole, to compute $\sin v$, $\cos v$, $\cos E$, $\sin u$, $\cos u$, $\log r$, of which the coefficients are very simple combinations. For this purpose different formulæ may be applied. Those to which I have been accustomed are the following:—

If the elements are given in the form suited to computation, and thence, also, the quantities for a certain time T, from which the perturbations are to be computed, referred to a fixed mean equinox, the same as that adopted for the disturbing planet, which quantities are,

L, the mean longitude of the disturbed planet,
μ, the mean daily sidereal motion,
π, the longitude of the perihelion,
φ, the angle, the sine of which is equal to the eccentricity,
Ω, the longitude of the ascending node,
i, the inclination,

then we compute in the first place the constants,

$a = \left(\frac{k}{\mu}\right)^{\frac{2}{3}}$, in which the $\log k$, since μ is given in seconds, here $=$ 3.5500066,

$e'' = \sin\varphi$ in seconds, or the logarithm of $206265 = 5.3144256$ added to $\log e$,

$$\left.\begin{array}{l}\sqrt{a\,(1-e)}\\ \sqrt{a\,(1+e)}\end{array}\right\}\begin{array}{l}\text{from which occurs the check, that}\\ \log\sqrt{a\,(1-e)} - \log\sqrt{a\,(1+e)} = \log\tan\left(45^\circ - \tfrac{1}{2}\varphi\right),\end{array}$$

$p = a\cos^2\varphi$,

and also the above-mentioned constants (1), (2), ... (13).

In order to combine all the constants, we can add here at once the reduction, necessary for the computation of the forces, of the orbit of the disturbing planet to that of the disturbed, or the computation of the quantities I, N, N', by means of the formulæ,

$$(20)\quad\begin{aligned}
\sin\tfrac{1}{2}I\sin\tfrac{1}{2}(N+N') &= \sin\tfrac{1}{2}(\Omega'-\Omega)\sin\tfrac{1}{2}(i'+i),\\
\sin\tfrac{1}{2}I\cos\tfrac{1}{2}(N+N') &= \cos\tfrac{1}{2}(\Omega'-\Omega)\sin\tfrac{1}{2}(i'-i),\\
\cos\tfrac{1}{2}I\sin\tfrac{1}{2}(N-N') &= \sin\tfrac{1}{2}(\Omega'-\Omega)\cos\tfrac{1}{2}(i'+i),\\
\cos\tfrac{1}{2}I\cos\tfrac{1}{2}(N-N') &= \cos\tfrac{1}{2}(\Omega'-\Omega)\cos\tfrac{1}{2}(i'-i),
\end{aligned}$$

where another check occurs in the agreement of $\sin\frac{1}{2}I$ and $\cos\frac{1}{2}I$.

If the computation of the perturbations first begins in general for T, it is necessary to calculate the places for $T-\frac{5}{2}\omega$, $T-\frac{3}{2}\omega$, $T-\frac{1}{2}\omega$, $T+\frac{1}{2}\omega$, ... $T+(i+\frac{1}{2})\omega$, if we wish to have the most convenient integration for the simple integrals that occur most frequently. With a suitable magnitude of the intervals, there is no occasion to go farther back than $T-\frac{5}{2}\omega$. If the slight inconvenience of the preparation of the first constants is not regarded, the places for $T-3\omega$, $T-2\omega$, $T-\omega$, T, $T+\omega$, ... $T+i\omega$ may also be calculated. But if the new computation for a later time T' joins on to a previous one, we seek first for the system of elements which, from the earlier computation, comes as near as possible to T', and calculate always with the new system the last place of the previous computation anew, in order to have the means of

ascertaining whether the elements in the former computation have not been regarded as constant for too long a time.

For the calculation of v, r, &c., the following formulæ answer then for every place: —

$$M = L - \pi + \mu\,(t - T);$$
$$M = E - e''\sin E;$$
$$\sin\tfrac{1}{2}v\sqrt{r} = \sin\tfrac{1}{2}E\sqrt{a\,(1+e)};$$
$$\cos\tfrac{1}{2}v\sqrt{r} = \cos\tfrac{1}{2}E\sqrt{a\,(1-e)}.$$

For the solution of the transcendental equation which gives E by means of M, let there be taken any, the nearest possible value of E . . . . . E', and compute

$$M' = E' - e''\sin E';$$

then is

$$M - M' = E - E' - e''\,(\sin E - \sin E')$$
$$= (E - E')\,(1 - e\cos E'),$$

provided $E' - E$ is regarded as a small quantity of the first order, and those of the second order are neglected for a time, so that a new approximate value of E is

$$E' + \frac{M - M'}{1 - e\cos E'},$$

with which we proceed in the same manner until the true value is obtained. It is almost always unnecessary to repeat the calculation of $1 - e\cos E'$. According to rule, if the first E' was not too much out of the way, the first computed value of $1 - e\cos E'$ might be retained in all the trials.

This process, which appears to be the most rapid and safe for the first approximation, is wholly identical with the rule of Gauss, to make use of the logarithmic difference with $\sin E$ . . . . . λ, ... and of the logarithmic difference with the number $e''\sin E$ . . . . . μ, ...

both naturally referred to the same unit, and then to adopt as the consequent approximate value

$$E' + (M - M') \cdot \frac{\mu}{\mu \pm \lambda}.$$

For, since λ is nothing else than

$$\lambda = \frac{d \log \sin E}{d E} = \frac{\cos E}{\sin E},$$

if we omit the modulus of Brigg's system, which afterwards disappears of itself, and

$$\mu = \frac{d \log (e'' \sin E)}{d (e'' \sin E)} = \frac{1}{e \sin E},$$

therefore is

$$\frac{\mu}{\mu - \lambda} = \frac{1}{1 - \frac{\lambda}{\mu}} = \frac{1}{1 - e \cos E},$$

so that the double sign $\mu \pm \lambda$ is always to be so taken as to agree with the sign of $\cos E$, if λ is always regarded as positive. Both formulæ would be wholly identical, if in practice the uncertainty of the logarithmic differences, in case the approximation is not yet very great, did not make the last form somewhat more doubtful than the first. The latter form, however, will be used with advantage in the last trial, in order to produce an agreement up to the last place of logarithms. Moreover, if several consecutive places are computed, the trials by differences formed from the previous results, and carried on for the new date, are so abridged, that the truth is found after four or five places almost without any trial. On this account, it will not be advisable to exhibit the trials in the actual computation. Only the final result, and its verification, need be given here.

The addition of the constant logarithms $\sqrt{a}\,(1 + e)$, &c., is made in the head by means of a paper, held over the other logarithms, on the lower border of which the constant logarithm stands; and so

in general with every combination of this sort of constants with variables.

The formation of the other quantities,

$$u = v + \pi - \Omega,$$

$$r \sin u,\ r \cos u,\ \sin v,\ \cos v,\ (\cos E + \cos v),\ \left(\frac{p}{r} + 1\right),\ r \sin v,$$

requires no explanation. The latter quantity, $r \sin v$, supplies a convenient check, because

$$r \sin v = a \cos \varphi \sin E,$$

and differs therefore by a constant logarithm from the $\log \sin E$ already written down.

If we wish to compute together $\frac{dM}{dt}$ and $\frac{dL}{dt}$ for the sake of verification, one value being otherwise sufficient, the Gaussian logarithms are not to be applied for

$$-(2 r \cos \varphi - p \cot \varphi \cos v),$$
$$-(2 r \cos \varphi + p \tan \tfrac{1}{2} \varphi \cos v),$$

nor for

$$-(10)\, r + (12) \cos v,$$
$$-(10)\, r - (9) \cos v,$$

but the numbers are to be sought in preference. If only one of the two values is computed, still the numbers can be made use of conveniently.

The appropriate combination of these values with the different constants (2) to (13) gives the logarithms of the coefficients of R_0, S_0, W_0, in the differential equations, by which also a portion of the fourth part is completed.

After this follows the third part, the computation of the forces, for which the requisite values N, N', I, have already been found. The collective formulæ are,

$$\sin\beta' = \sin(L' - (\Omega' + N'))\sin I,$$
$$\tan\lambda' = \tan(L' - (\Omega' + N'))\cos I,$$

where $\cos\lambda'$ must always have the same sign as $\cos(L' - (\Omega' + N'))$,

$$z' = r'\sin\beta',$$
$$y' = r'\cos\beta'\sin(\lambda' - (v + \omega')),$$
$$x' = r'\cos\beta'\cos(\lambda' - (v + \omega')),$$

in which $\omega' = \pi - \Omega - N,$

and furthermore, $\cos\beta'\cos(\lambda' - (v + \omega')) = \cos\gamma',$

$$r'\sin\gamma' = \varrho\sin l',$$
$$r - r'\cos\gamma' = r - x' = \varrho\cos l',$$

without actually taking out the angles γ' and l'. Then,

$$\frac{1}{\varrho^3} - \frac{1}{r'^3} = \varDelta,$$

in which the Gaussian logarithms are used with great advantage. Finally,

$$W_0 = (1)\,\varDelta z', \qquad S_0 = (1)\,\varDelta y', \qquad R_0 = (1)\left\{\varDelta x' - \frac{r}{\varrho^3}\right\}.$$

These forces must be combined with the appropriate coefficients; namely, for

$di \ldots r\cos u$ . . . . with W_0,

$d\Omega \ldots (2)\, r\sin u$ . . . . . . " W_0,

$d\varphi \ldots \begin{cases} (3)\sin v & \text{. . . . . . " } R_0, \\ (3)(\cos v + \cos E) & \text{. . . . . " } S_0, \end{cases}$

$d\pi \ldots \begin{cases} -(4)\cos v & \text{. . . . . . " } R_0, \\ +(5)\left(\frac{\varrho}{r} + 1\right) r\sin v & \text{. . . . " } S_0, \\ +(6)\, r\sin u & \text{. . . . . " } W_0, \end{cases}$

$d\mu \ldots \begin{cases} -(7)\sin v & \text{. . . . . . " } R_0, \\ -(8)\frac{1}{r} & \text{. . . . . . " } S_0, \end{cases}$

$$dL\,..\begin{cases} -(9)\cos v-(10)\,r & \text{with } R_0, \\ +(11)\left(\frac{p}{r}+1\right) r\sin v & \text{" } S_0, \\ +(6)\,r\sin u & \text{" } W_0, \end{cases}$$

$$dM\begin{cases} +(12)\cos v-(10)\,r & \text{" } R_0, \\ -(13)\left(\frac{p}{r}+1\right) r\sin v & \text{" } S_0. \end{cases}$$

The double integral

$$\iint\frac{d\mu}{dt}\,dt^2$$

of L and M is added separately after the integration. In this manner are given the numerical values of the differential coefficients, copious examples of the integration of which are contained in the treatise upon mechanical quadratures. The constants to be added for the elements i, Ω, π, φ, are the original adopted values for a fixed time T, and the integration gives immediately the increase, since the factor ω is contained in (1). The first integral gives for μ, $\omega\,\Delta\,\mu$, because the quadratic factor ω^2 appears in the differential coefficient; ONE multiplication by ω is affected by means of the constant (1), the OTHER by (7) and (8). For L and M, there is still to be added $L_0+\mu_0\,(t-T)$ and $M_0+\mu_0\,(t-T)$, besides the double integral, and the result of the integration of $\frac{dL}{dt}$. Finally, it must not be forgotten that all longitudes are referred to a fixed mean equinox, and therefore the precession and nutation must be applied for any other epoch.

So far as the accuracy required in this computation is concerned, logarithms of five places of decimals appear to be sufficient. If the perturbations should be so great, that these are not satisfactory for a fixed interval, it would be expedient, for other reasons, so to reduce the interval that the five decimal places would still afford all that is desired. They are also to be taken in preference, because with them the computation can be carried through very rapidly. With

some effort, the complete computation of the perturbations, for about ten intervals, can be finished in one day, even if, which may be advisable as a general rule, v, r, and all the constants are calculated with six places of decimals, in order to be certain of the final fifth place, and in order not to obtain in the solution of the transcendental equation for E results which, sufficiently accurate in themselves, might still want conformity in the differences by means of which a sure proof of the correctness of the computation is obtained. This verification by means of differences should not be neglected, particularly at the end. The fitting on of the later computation to the previous one affords a security against constant errors; the regularity of the differences, against accidental errors.

In the following detailed example, in which no single number is wanting that was incidentally computed, with the exception of the inconsiderable calculation of Ω' and i', and the trials for E, in which the comparatively small tediousness and difficulty of this computation are plainly evident, I have, for the convenience of printing, carried out the calculation with four decimals.

The comparison of the results obtained with five decimals with these shows that in ordinary cases four decimals will be sufficient. Still, the saving of time does not appear to me to be important enough to sacrifice to it the advantage of obtaining the angles accurate within a few seconds, which is possible with five decimals, while with four decimals parts of minutes only are given.

The previously established elements for June 12, 1836, derived from initial values resulting from a former computation, will answer for an example of the integration.

PERTURBATIONS OF VESTA BY JUPITER.

1836, JULY 3—DECEMBER 18.

PLACES OF JUPITER.

The interpolation from the Almanac gives for the apparent place:—

1836, 0h. P. M. T.	Longitude ♃	Latitude ♃	Radius Vector.
July 3	115° 58′ 35″.8	+0° 23′ 19″.9	5.25433
August 14	119 23 6.1	27 45.6	5.26916
September 25	122 46 26.2	32 4.0	5.28371
November 6	126 8 40.4	36 14.3	5.29794
December 18	129 29 53.1	40 16.1	5.31179

The subtraction of the nutation and precession since 1810, January 0, in order to bring everything to the mean equinox of this epoch, gives the reduced longitudes,—

1836, 0h. P. M. T.	$t - 1810$.	Correction for Nutation.	Correction for Precession.	Reduced Longitudes.
July 3	9681	+11″.6	—22′ 11″.3	115° 36′ 36″.1
August 14	9723	10.4	22 17.1	119 0 59.4
September 25	9765	11.3	22 22.9	122 24 14.6
November 6	9807	12.1	22 28.7	125 46 23.8
December 18	9849	10.4	22 34.5	129 7 29.0

From the places,

July 3	115° 36′ 36″.1	+0° 23′ 19″.9
December 18	129 7 29.0	40 16.1

we have, $\Omega' = 98°\ 22'\ 56''.0$ $\qquad i' = 1°\ 18'\ 45''.9$;

whence the reduction to the longitudes in the orbit is

$$+27''.073 \sin 2\ (\text{reduced longitudes} - \Omega').$$

Consequently we have

1836, 0h. P. M. T.	Reduct.	L'	Log r'
July 3	+15″.3	115° 36′ 51″.4	0.720517
August 14	17.9	119 1 17.3	0.721741
September 25	20.1	122 24 34.7	0.722939
November 6	22.1	125 46 45.9	0.724107
December 18	23.8	129 7 52.8	0.725241

ELEMENTS OF VESTA.

1836, June 12, 0^h Paris Mean Time.

$M = 319^\circ\ 13'\ 1''.8$	$\pi = 250^\circ\ 7'\ 56''.4$
$\mu = 977''.83172$	$\Omega = 102\ 59\ 1.6$
$\varphi = 5^\circ\ 2'\ 33''.6$	$i = 7\ 8\ 15.7$

2.990264	8.943977	9.998316	0.087898
3.550007	5.314425	9.996632	0.036587
0.559743	4.258402	0.369794	0.373162
0.186581	$\log e''$	$\log p$	9.960043
0.373162			$\log\sqrt{a}\,(1+e)$ . . . 0.204875
$\log a$			$\log\sqrt{a}\,(1-e)$ . . . 0.166603

$$\log\frac{42\,k}{\sqrt{p}} = 9.673934 \qquad \log\frac{126\,k}{\sqrt{a}} = 0.149371$$

$\log m' = 2.29161\bar{6}$ Jupiter's mass in seconds.

$\log (1) \ldots \frac{42\,k\,m'}{\sqrt{p}} \ldots$ 1.965550	$\log (7) \ldots \frac{126\,k}{\sqrt{a}}\,e \ldots$ 9.093348
$\log (2) \ldots \frac{1}{\sin i} \ldots$ 0.905688	$\log (8) \ldots \frac{126\,k}{\sqrt{a}}\,p \ldots$ 0.519165
$\log (3) \ldots a\cos\varphi \ldots$ 0.371478	$\log (9) \ldots p\tan\frac{1}{2}\varphi$ 9.013582
$\log (4) \ldots \frac{p}{e} \ldots$ 1.425817	$\log (10) \ldots 2\cos\varphi$ 0.299346
$\log (5) \ldots \frac{1}{e} \ldots$ 1.056023	$\log (11) \ldots \tan\frac{1}{2}\varphi$ 8.643788
$\log (6) \ldots \tan\frac{1}{2}i \ldots$ 8.794967	

$\Omega' \ldots 98^\circ\ 22'\ 56''.0$	$i' \ldots 1^\circ\ 18'\ 45''.9$	$\frac{1}{2}(\Omega' - \Omega) \ldots 177^\circ\ 41'\ 57''.2$
$\Omega \ldots 102\ 59\ 1.6$	$i \ldots 7\ 8\ 15.7$	$\frac{1}{2}(i' + i) \ldots$ 4 13 30.8
		$\frac{1}{2}(i' - i) \ldots$ −2 54 44.9

8.867333	8.705952_n	$\frac{1}{2}(N + N') = 3^\circ\ 20'\ 3''.3$
8.603635	9.999650_n	$\frac{1}{2}(N - N') = 177\ 42\ 9.0$
9.998818	9.999439	$N \ldots$ 181 2 12.3
7.470968	8.602453	$\pi - \Omega \ldots$ 147 8 54.8
8.705602	9.999089_n	$\omega' \ldots$ 326 6 42.5
9.999264	9.999651	$N' \ldots$ 185 37 54.3
8.706338	9.999438	$\Omega' \ldots$ 98 22 56.0
$\frac{1}{2}I \ldots 2^\circ\ 54'\ 54''.2$	$I \ldots 5^\circ\ 49'\ 48''.4$	$\Omega' + N' \ldots$ 284 0 50.3

$\log\sin I$ . . . 9.006804 $\quad\log\cos I$ . . . 9.997747.

DISTURBED PLANET . . . VESTA.

1836, 0h. P. M. T.	July 3.	August 14.	September 25.	November 6.	December 18.
M	324° 55'.3	336° 19'.8	347° 44'.2	359° 8'.7	10° 33'.2
E	321 48.4	334 8.0	346 34.0	359 3.8	11 33.8
log sin E	9.7913_n	9.6397_n	9.3661_n	8.2134	9.3020
e sin E	—3 6.9	—2 11.8	—1 10.2	—0 4.9	+1 0.6
$\frac{1}{2}E$	160 54.2	167 4.0	173 17.0	179 31.9	5 46.9
log sin $\frac{1}{2}E$	9.5147	9.3499	9.0680	7.9124	9.0032
log cos $\frac{1}{2}E$	9.9754_n	9.9888_n	9.9970_n	0.0000_n	9.9978
log sin $\frac{1}{2}v\sqrt{r}$	9.7196	9.5548	9.2729	8.1173	9.2081
log cos $\frac{1}{2}v\sqrt{r}$	0.1420_n	0.1554_n	0.1636_n	0.1666_n	0.1644
$\frac{1}{2}v$	159 17.4	165 55.1	172 40.3	179 29.3	6 18.6
cos $\frac{1}{2}v$	9.9710_n	9.9867_{5n}	9.9964_n	0.0000_n	9.9973
log $\sqrt{r}$	0.1710	0.1686_5	0.1672	0.1666	0.1671
v	318 34.8	331 50.2	345 20.6	358 58.6	12 37.2
log r	0.3420	0.3373	0.3344	0.3332	0.3342
u	105 43.7	118 59.1	132 29.5	146 7.5	159 46.1
log cos u	9.4331_n	9.6854_n	9.8296_n	9.9192_n	9.9723_n
log sin u	9.9835	9.9419	9.8677	9.7462	9.5388
log r sin u	0.3255	0.2792	0.2021	0.0794	9.8730
log sin v	9.8206_n	9.6740_n	9.4032_n	8.2519_n	9.3394
log cos v	9.8750	9.9453	9.9856	9.9999	9.9894
log cos E	9.8953	9.9542	9.9879	9.9999	9.9911
	0.2911	0.2966	0.2999	0.3010	0.3002
log (cos v + cos E)	0.1864	0.2508	0.2878	0.3009	0.2913
log $\frac{p}{r}$	0.0278	0.0325	0.0354	0.0366	0.0356
log $\left(\frac{p}{r}+1\right)$	0.3152	0.3176	0.3191	0.3197	0.3192
log r sin v	0.1626_n	0.0113_n	9.7376_n	8.5851_n	9.6736
log $(p+r)$ sin v	0.4778_n	0.3289_n	0.0567_n	8.9048_n	9.9928
—p tan $\frac{1}{2}\varphi$ cos v	—0.0774	—0.0910	—0.0998	—0.1032	—0.1007
—2 r cos φ	—4.3780	—4.3310	—4.3020	—4.2900	—4.3000
log Coeff. dL	0.6489_n	0.6456_n	0.6437_n	0.6428_n	0.6436_n

DISTURBING PLANET . . . JUPITER.

1836, 0h. P. M. T.	July 3.	August 14.	September 25.	November 6.	December 18.
u'	$191^\circ\ 36'.0$	$195^\circ\ 0'.4$	$198^\circ\ 23'.7$	$201^\circ\ 45'.9$	$205^\circ\ 7'.0$
$\log \sin u'$	9.3034_n	9.4132_n	9.4991_n	9.5691_n	9.6278_n
$\log \tan u'$	9.3123	9.4283	9.5219	9.6013	9.6710
λ'	191 32.3	194 55.9	198 18.3	201 39.7	205 0.0
$v + \omega'$	284 41.5	297 56.9	311 27.3	325 5.3	338 43.9
$\lambda' - (v + \omega')$	266 50.8	256 59.0	246 51.0	236 34.4	226 16.1
$\log \sin \beta'$	8.3102_n	8.4200_n	8.5059_n	8.5759_n	8.6346_n
$\log \cos [\lambda' - (v + \omega')]$	8.7405_n	9.3526_n	9.5945_n	9.7411_n	9.8396_n
$\log \cos \beta'$	9.9999	9.9999	9.9998	9.9997	9.9996
$\log \sin [\lambda' - (v + \omega')]$	9.9994_n	9.9887_n	9.9636_n	9.9215_n	9.8589_n
$\log \frac{y'}{r'}$	9.9993_n	9.9886_n	9.9634_n	9.9212_n	9.8585_n
$\log \cos \gamma'$	8.7404_n	9.3525_n	9.5943_n	9.7408_n	9.8392_n
$\log \sin \gamma'$	9.9994	9.9887	9.9636	9.9216	9.8593
$\log r$	0.3420	0.3373	0.3344	0.3332	0.3342
$\log x'$	9.4609_n	0.0742_n	0.3172_n	0.4649_n	0.5644_n
	0.0537	0.1891	0.2926	0.2402	0.2010
$\log \varrho \cos l'$	0.3957	0.5264	0.6270	0.7051	0.7654
$\log \varrho \sin l'$	0.7199	0.7104	0.6865	0.6457	0.5845
$\log \cos l'$	9.9560	9.9226	9.8772	9.8772	9.9216
$\log \varrho$	0.7639	0.7878	0.8093	0.8279	0.8438
$\log \frac{1}{\varrho^3}$	7.7083	7.6366	7.5721	7.5163	7.4686
$\log \left(-\frac{1}{r'^3}\right)$	7.8384_n	7.8348_n	7.8312_n	7.8277_n	7.8243_n
	0.5870	0.4360	0.3475	0.2910	0.2525
$\log \varDelta$	7.2514_n	7.3988_n	7.4837_n	7.5367_n	7.5718_n
$\log y'$	0.7198_n	0.7103_n	0.6863_n	0.6453_n	0.5837_n
$\log (1)\ \varDelta$	9.2169_n	9.3643_n	9.4492_n	9.5022_n	9.5373_n
$\log z'$	9.0307_n	9.1417_n	9.2288_n	9.3000_n	9.3598_n
$\log \varDelta x'$	6.7123	7.4730	7.8009	8.0016	8.1362
$\log \left(-\frac{r}{\varrho^3}\right)$	8.0503_n	7.9739_n	7.9065_n	7.8495_n	7.8028_n
	0.0204	0.1647	0.6656	0.5294	0.2709
$\log R'$	8.0299_n	7.8092_n	7.2409_n	7.4722	7.8653

FORCES AND VARIATIONS OF THE ELEMENTS.

1836, 0h. P. M. T.		July 3.	August 14.	September 25.	November 6.	December 18.
$\log R_0$	$\frac{42\,k}{\sqrt{p}}$	9.9954$_n$	9.7747$_n$	9.2064$_n$	9.4377	9.8308
$\log S_0$		9.9367	0.0746	0.1355	0.1475	0.1210
$\log W_0$		8.2476	8.5060	8.6780	8.8022	8.8971
$d\,i$	W	9.7751$_n$	0.0227$_n$	0.1640$_n$	0.2524$_n$	0.3065$_n$
$d\,\Omega$	W	1.2312	1.1849	1.1078	0.9851	0.7787
$d\,\varphi$	R	0.1921$_n$	0.0455$_n$	9.7747$_n$	8.6234$_n$	9.7109
	S	0.5579	0.6223	0.6593	0.6724	0.6628
$d\,\pi$	R	1.3008$_n$	1.3711$_n$	1.4114$_n$	1.4257$_n$	1.4152$_n$
	S	1.5338$_n$	1.3849$_n$	1.1127$_n$	9.9608$_n$	1.0488
	W	9.1205	9.0742	8.9971	8.8744	8.6680
$d\,\mu$	R	8.9139	8.7673	8.4965	7.3452	8.4327$_n$
	S	0.1772$_n$	0.1819$_n$	0.1848$_n$	0.1860$_n$	0.1850$_n$
$d\,L$	R	0.6489$_n$	0.6456$_n$	0.6437$_n$	0.6428$_n$	0.6436$_n$
	S	9.1216$_n$	8.9727	8.7005$_n$	7.5486$_n$	8.6366
	W	9.1205	9.0742	8.9971	8.8744	8.6680
$42\,d\,i$		− 0″.011	− 0″.034	− 0″.070	− 0″.113	− 0″.160
$42\,d\,\Omega$		+ 0.301	+ 0.491	+ 0.611	+ 0.613	+ 0.474
		+ 1.540	+ 0.661	+ 0.096	− 0.012	+ 0.348
		+ 3.123	+ 4.976	+ 6.234	+6.606	+ 6.079
$42\,d\,\varphi$		+ 4.663	+ 5.637	+ 6.330	+6.594	+ 6.427
		+19.777	+13.990	+ 4.148	−7.302	−17.620
		−29.547	−28.807	−17.708	−1.283	+14.783
		+ 0.002	+ 0.004	+ 0.005	+0.005	+ 0.004
$42\,d\,\pi$		− 9.768	−14.813	−13.555	−8.580	− 2.833
		− 0.0811	− 0.0348	− 0.0050	+0.0006	− 0.0183
		− 1.3000	− 1.8050	− 2.0910	−2.1552	− 2.0229
$(42)^2\,d\,\mu$		− 1.3811	− 1.8398	− 2.0960	−2.1546	− 2.0412
		+ 4.409	+ 2.632	+ 0.708	−1.204	− 2.981
		− 0.114	− 0.111	− 0.069	−0.005	+ 0.057
		+ 0.002	+ 0.004	+ 0.005	+0.005	+ 0.004
$42\,d\,L$		+ 4.297	+ 2.525	+ 0.644	−1.204	− 2.920

FOR THE INTEGRATION.

$i = 7° \; 8' \; 11''.64$ $\qquad$ $\Omega = 103° \; 8' \; 20''.48$

1836, 0h. P. M. T.	$t-1810.$	$42\,d\,i$	$\Delta\,i$	$42\,d\,\Omega$	$\Delta\,\Omega$
May 22	9639	000″.0		+0″.102	
			+4″.083		−558″.916
July 3	9681	−0.011		+0.301	
			+4.072		−558.615
August 14	9723	−0.034		+0.491	
			+4.038		−558.124
September 25	9765	−0.070		+0.611	
			+3.968		−557.513
November 6	9807	−0.113		+0.613	
			+3.855		−556.900
December 18	9849	−0.160		+0.474	

$\varphi = 5° \; 9' \; 39''.17$ $\qquad$ $\pi = 249° \; 48' \; 26''.91$

1836, 0h. P. M. T.	$t-1810.$	$42\,d\,\varphi$	$\Delta\,\varphi$	$42\,d\,\pi$	$\Delta\,\pi$
May 22	9639	+3″.624		+ 3″.886	
			−425″.567		+1170″.016
July 3	9681	+4.663		− 9.768	
			−420.904		+1160.248
August 14	9723	+5.637		−14.813	
			−415.267		+1145.435
September 25	9765	+6.330		−13.555	
			−408.937		+1131.880
November 6	9807	+6.594		− 8.580	
			−402.343		+1123.300
December 18	9849	+6.427		− 2.833	

$\mu = 978''.29671$ $\qquad$ $L = 105° \; 53' \; 15''.63$

1836, 0h. P. M. T.	$t-1810.$	$(42)^2\,d\,\mu$	$42\,\Delta\,\mu$	$\Delta\,M$	$42\,d\,L$	$\Delta\,L$
May 22	9639	−0″.7115		−4802″.3339	+5″.825	
			−19″.5017			−1071″.572
July 3	9681	−1.3811		−4821.8356	+4.297	
			−20.8828			−1067.275
August 14	9723	−1.8398		−4842.7184	+2.525	
			−22.7226			−1064.750
September 25	9765	−2.0960		−4865.4410	+0.644	
			−24.8186			−1064.106
November 6	9807	−2.1546		−4890.2596	−1.204	
			−26.9732			−1065.310
December 18	9849	−2.0412		−4917.2328	−2.920	

The constant elements belong to January 0, 1810, Paris mean time. The longitudes are referred to the mean equinox of the same epoch.

[The Method of Integration, &c. will be given in the next Number.]

Mathematical Monthly Notices.

The Mathematical Correspondent, containing new Elucidations, Discoveries, and Improvements in various branches of the Mathematics, with collections of Mathematical questions resolved by ingenious Correspondents, adapted to the present state of learning in America, and designed to inspire youth with the love of Mathematical knowledge by alluring their attention to the solutions of pleasant and curious questions, and to promote the cultivation of the Mathematics by opening a channel for the ready conveyance of discoveries and improvements from one mathematician to another. "In the mathematical sciences truth appears most conspicuous and shines in its greatest lustre." — EMERSON. Edited by GEORGE BARON. In quarterly numbers of one sheet each. Vol. I. pp. 248. New York. Printed for the Editor by Sage and Clough. 1804. This volume is in the Astor Library, New York.

The Analyst; or, Mathematical Museum, containing new Elucidations, Discoveries, and Improvements in various branches of the Mathematics, with collections of questions proposed and resolved by ingenious Correspondents. Conducted by R. ADRAIN, A. M., Fellow of the American Academy of Arts and Sciences, of the American Philosophical Society, of the Literary and Philosophical Society of New York, and Professor of Mathematics and Natural Philosophy in Columbia College, New York. Nos. I.–V. pp. 150. Philadelphia: Published by William P. Farrand. 1808. No. I. New York: Printed and Published by George Long, 71 Pearl Street. 1814.

The Scientific Journal. Conducted by MR. MARRAT. Published by J. T. Murden & Co. Perth Amboy, N. J. It appears, from a volume of this work in the Astor Library, that it was published in numbers in 1818 and 1819. It contains nine numbers, February to September, 1819, and July and October, 1819. pp. 184. Either this is an imperfect volume, or the numbers were not regularly published.

The Philosophic Magazine, or Gentleman's Diary. Edited by MR. NASH. We have failed to find more than this simple announcement.

The Mathematical Diary, containing new researches and improvements in the Mathematics, with collections of questions proposed and resolved by ingenious Correspondents. In quarterly numbers. Conducted by R. ADRAIN, LL. D., F. A. P. S., F. A. A. S., &c., and Professor of Mathematics and Natural Philosophy in Columbia College, New York. Nos. I.–VIII. Vol. I. pp. 316. 1825. Nos. I.–II. Vol. II. pp. 108. 1828.

The Mathematical Miscellany. Conducted by C. GILL, Professor of Mathematics in the Institute at Flushing, Long Island. Published at the Institute. New York: W. E. Dean, Printer, No. 2 Ann Street. 1836.

This Periodical was published semiannually. The first Number appeared in February, 1836, and the eighth and last on November 1, 1839. Nos. I.–VI. constitute Vol. I. of 414 pages. Nos. VII. and VIII. contain 142 additional pages.

The Cambridge Miscellany of Mathematics, Physics, and Astronomy. To be continued quarterly. Edited by BENJAMIN PEIRCE, A. A. S., Perkins Professor of Astronomy and Mathematics in Harvard University; and JOSEPH LOVERING, A. A. S., Hollis Professor of Mathematics and Natural Philosophy in Harvard University. Nos. I.–IV., from July, 1842, to January, 1843. pp. 192. 8vo.

The above are all the facts we have been able to gather in regard to the publication of these mathematical and scientific serials; and we publish them in this incomplete state in hopes that some of our correspondents may be induced to revise them, and give the readers of the Monthly a correct account of these early and interesting periodicals.

Editorial Items.

THE following gentlemen have sent us solutions of the Prize Problems in the December number of the Monthly: —

M. K. BOSWORTH, Sophomore Class, Marietta College, Ohio, Probs. III., IV., and V.
WILLIAM HINCHCLIFFE, Barre Plains, Mass., Probs. I., II., III., IV., and V.; but III. and IV. came too late for the Committee.
DAVID TROWBRIDGE, Perry City, N. Y., Probs. III., IV., and V.
J. Q. HOLLISTER, Hamilton College, Clinton, N. Y., Probs. III., IV., and V.
J. A. WINEBRENER, Princeton College, N. Y., Probs. I., II., III., IV., and V.
WILLIAM MINTO, University of Michigan, Probs. III., IV., and V.
C. A. BUCKINGHAM, Hamilton College, N. Y., Probs. III., IV., and V.
H. TIEMAN, Baltimore, Md., Probs. I., II., III., IV., and V.
CORYDON C. OLNEY, Nunda Literary Inst., N. Y., Probs. I. and II.
ARTHUR M. CAZIMAJOU, Polytechnic College, Philadelphia, Prob. V.
S. E. BENJAMIN, Patten, Me., Prob. III.
JAMES F. ROBERSON, Senior Class, Indiana University, Prob. V.
ASHER B. EVANS, Madison University, Hamilton, N. Y., Probs. III., IV., and V.
F. E. TOWER, Senior Class, Amherst College, Probs. III., IV., and V.
WARREN PHELPS, Cortlandville, N. Y., Prob. V.
WILLIAM REYNOLDS, University of Maryland, Baltimore, Probs. I., II., III., IV., and V.
FRANK N. DEVEREUX, Boston, Mass., Probs. I. and II.
GUSTAVUS FRANKENSTEIN, Springfield, Ohio, Probs. III. and V.

Books Received. — Report on Weights and Measures, read before the Pharmaceutical Association at their Eighth Annual Session, held in Boston, September 15, 1859. By Alfred B. Taylor, of Philadelphia, Chairman of the Committee on Weights and Measures. Boston: Press of George C. Rand & Avery, No. 3 Cornhill. 1859. . . . . . *Physical Optics.* Part II. The Corpuscular Theory of Light discussed Mathematically. By RICHARD POTTER, A. M., F. C. P. S., &c. Cambridge: Deighton, Bell, & Co. London: Bell & Dalby. 1859. . . . . . *Of Motion.* An Elementary Treatise. By JOHN ROBERT LUNN, A. M., Fellow and Lady Sadlier's Lecturer of St. John's College. Cambridge: Deighton, Bell, & Co. London: Bell & Dalby. 1859. . . . . . Address before the American Association for the Advancement of Science, August, 1859. By Professor ALEXIS CASWELL. Published for the Association by JOSEPH LOVERING, Permanent Secretary. 1859. . . . . . Annual Report of the Board of Regents of the Smithsonian Institution, showing its operations, expenditures, and condition for the year 1858. Washington: James B. Stedman, Printer. 1859. . . . . . The Surveyor's Companion, containing a Treatise on Mathematical Instruments, &c., &c. By WILLIAM SCHMOLZ, Mathematical-Instrument Maker, San Francisco, California. 1859. A new set of Practical Tables, useful in Surveying and Engineering, &c.; together with an improved method of Tabling, which facilitates the computation of areas and the projection of Maps. By R. C. MATHEWSON, U. S. Deputy Surveyor. Published by WILLIAM SCHMOLZ, San Francisco. 1859. . . . . . An Elementary Treatise on Hydrostatics, for the use of Junior University Students. By RICHARD POTTER, A. M., F. C. P. S., late Fellow of Queen's College, Cambridge, Licentiate of the Royal College of Physicians, London, Honorary Member of the Literary and Philosophical Society of St. Andrews; Professor of Natural Philosophy and Astronomy in University College, London. Cambridge: Deighton, Bell, & Co. London: Bell & Dalby. 1859.

March, 1860.

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"Late State Superintendent of Schools, Indiana."

LYMAN C. DRAPER, Esq., Superintendent of Public Instruction of Wisconsin.

Hon. J. M. GREGORY, Superintendent of Public Instruction for Michigan, and

Hon. JONATHAN TENNEY, Secretary Board of Education in New Hampshire, have also recommended *Robinson's Mathematics.*

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Rev. CHESTER DEWEY, D.D., Professor in the Rochester University, N. Y., writes: — "I have perused both your Astronomy and Algebra with great pleasure. They are admirable works of the kind, and no one can fail to give them approbation and high commendation."

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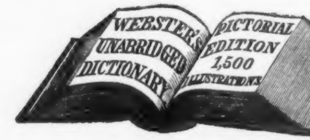

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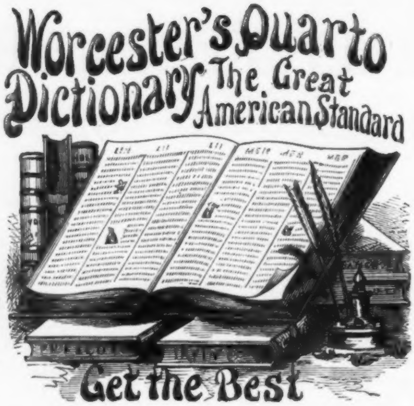

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THE MATHEMATICAL MONTHLY.

Edited by J. D. RUNKLE, A.M., A.A.S., Cambridge, Mass.

THIS is an Educational Journal devoted, 1. to the elevation of the standard of Mathematical Learning in this country, and 2. to the advancement of the Science.

Its plan and aims, as set forth in the first number of Volume I., have received the hearty approval of a large proportion of the MATHEMATICIANS, ASTRONOMERS, ENGINEERS, and SCIENTIFIC MEN in the United States and British Provinces, the American Association for the Advancement of Science, and several State Teachers' Associations. The first volume of the Monthly contains the names of not less than three hundred of those who have given it their cordial approbation.

For the information of those whose attention has not before been called to the subject, the Publishers beg to offer the following selections from a large number of testimonials; having room only to refer to notices in the Atlantic Monthly, Silliman's Journal, Journal of the Franklin Institute, Christian Examiner, Nouvelles Annales de Mathématiques, and nearly all the educational journals and prominent newspapers in all parts of the country.

After the issue of the eighth number, the following card was issued: —

"BOSTON, May 25th, 1859.

"The undersigned having watched with great interest the establishment of a new MATHEMATICAL JOURNAL at Cambridge, under the editorship of Mr. J. D. RUNKLE, are desirous of calling the attention of the patrons of sound learning to this work. The 'MATHEMATICAL MONTHLY' commenced its existence in October, 1858, and has been conducted on a plan that cannot fail to make it an instrument of great good. It is addressed to students as well as to professors; and has doubtless already given a new impulse to mathematical studies, wherever it has been introduced. A work of this kind should not be suffered simply to live. It has now about eleven hundred subscribers; and is not, perhaps, likely to be altogether discontinued, while its present support remains. But its friends should not be satisfied with this. A liberal subscription should insure to it a vigorous, energetic, long-continued life; and should enable it not only to preserve its present excellent form, and the stimulus of its prizes, — now amounting to about three hundred dollars, — but to make new improvements; to increase its size, without increasing its terms; to secure the best matter, by paying contributors, if necessary; and to compensate its editor, in part at least, for the time he bestows upon it.

"This appeal is made from no suggestion of the editor, but from the conviction that the work deserves a wider patronage, and must secure it, in order to be *permanently* successful. The circle of strictly mathematical readers in this country is yet small. To enlarge it, nothing can be more surely relied on than a well-conducted Journal; but until this is done, and the work is thus made to support itself, the aid of *all* true friends of Science must be invoked.

"JAMES WALKER,
BENJAMIN PEIRCE,
G. P. BOND,
EDWARD EVERETT,
JARED SPARKS,
JOSEPH LOVERING,
JOSIAH QUINCY,
J. INGERSOLL BOWDITCH."

The Lady's and Gentleman's Diary for 1860, edited by Professor WOOLHOUSE, London, England, says of the Mathematical Monthly: — "This interesting American monthly periodical has completed its first volume. It is both ably conducted and efficiently supported, and gives an earnest of success that must be gratifying to its promoters, and satisfactory to its numerous talented contributors."

Professor HENRY, in his last Smithsonian Report, says: — "A subscription has also been made for a number of copies, for foreign distribution, of the Mathematical Monthly, edited by J. D. RUNKLE, of Cambridge, a journal intended to promote the study of mathematics in this country. The plan and execution of this work are such as to commend it to all who are interested in the advance of the important branch of knowledge to which it pertains; and it is gratifying to learn that the patronage which it has received, as well as the number and character of the articles furnished, are such as to insure its entire success."

Professor WILLIAM RUTHERFORD of the Royal Military Academy, Woolwich, England, in a note to the editor, says: — "I have also got all the numbers of your new Monthly Mathematical publication, with which I am much pleased. It will be a useful work, and I regret we have not any similar publication in Britain."

From hundreds of letters we might compile a volume of strongest testimonials to the merits and importance of the MONTHLY.

The Publishers will keep up the size, style, and beauty of its typography and illustrations, which have been distinguished for their artistic excellence. Two or more steel plates, fully equal to those of DONATI'S Comet and the admirable portrait of Dr. BOWDITCH, which has already appeared, will be given in each volume, with as many woodcuts as shall be necessary fully to illustrate the text.

Mr. EVERETT says, in speaking of the appearances of the comet as seen through the great Refractor of Harvard College Observatory: "They are illustrated by two admirable engravings on steel, from drawings executed from sketches taken by the side of the great Refractor, one by Mr. GEORGE P. BOND, and the other by Mr. FETTE. Both are beautiful; but the former appears to me the most admirably executed work of the kind I have ever seen; not only far beyond any European drawing or engraving of this comet which has yet reached us, but superior to any foreign drawings and engravings of any celestial phenomena; those, for instance, in Sir JOHN HERSCHEL'S splendid work, 'The Results of Astronomical Observations at the Cape of Good Hope.' The drawings and engravings in that fine volume, though executed at the expense of a munificent patron (the Duke of Northumberland), and by the most skilful English artists, are inferior to the drawings of Messrs. BOND and FETTE, engraved by J. W. WATTS at Boston, for the Mathematical Monthly."

We shall soon present our readers with a fine portrait on steel of Professor PEIRCE of Harvard University, which we hope to follow by one of the distinguished LEVERRIER.

The Monthly will contain frequent lists of Foreign and American Mathematical and Scientific Publications, with notices and reviews of those most likely to be useful to the teachers and students of mathematics.

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